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Prediction of Pore Water Pressures in Several Embankment Dams by Normal and Back Analyses

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Abstract: This paper describes the prediction of seepage in the impervious zones of six embankment dams by seepage analysis. In addition, by making use of the available monitored pore water pressure data of the first filling of reservoir, back analysis was carried out to estimate the best fit seepage parameters. The estimated parameters were used for further prediction of pore water pressures in subsequent monitored durations with reasonable accuracy. The accuracy of prediction was evaluated quantitatively by the prediction difference and error index. The results show that the error indices in the upstream sides of the impervious zones are smaller than those in the downstream sides.

Key words: *Unsaturated soils, fill dams, pore water pressure, back analysis, FEM*

1. Introduction

Every dam must be subjected to the filling test to ensure its safety performance. For an embankment dam, it is imperative to monitor the behavior of its pore water pressures particularly in the early stage of its service life span of first reservoir filling. However, the safety of the dam is not proven until it is subjected to the first filling to its top water level. The prediction of pore water pressures in fill dams in the early stage of first filling reservoir particularly poses some difficulty as the embankment fill is in unsaturated condition immediately after construction. This paper describes the prediction of pore water pressures in embankment dams, here known as A, B, C, D, E and F Dams, subjected to variable head conditions in the early stages of reservoir fluctuations. The dams, which are located in the Prefectures of Okayama and Hiroshima, Western Japan, were constructed for the purposes of irrigation and flood control in the rural areas. The characteristics of these dams, the number of electric piezometers installed and the characteristics of the fills are as shown in Table 1. The representative cross-section of each dam is as shown in Figure 1. The Properties of the fills are mainly SM, GM, SC in the Unified Soil Classification System with maximum densities of 1.54-1.79 (g/cm^3) and optimum water content of 14.9-30.2%.

2. Methods of Analyses for Prediction of Pore Pressures

2.1 Normal Analysis of Seepage

In the present analysis, numerical calculations are carried out by the finite element method. The

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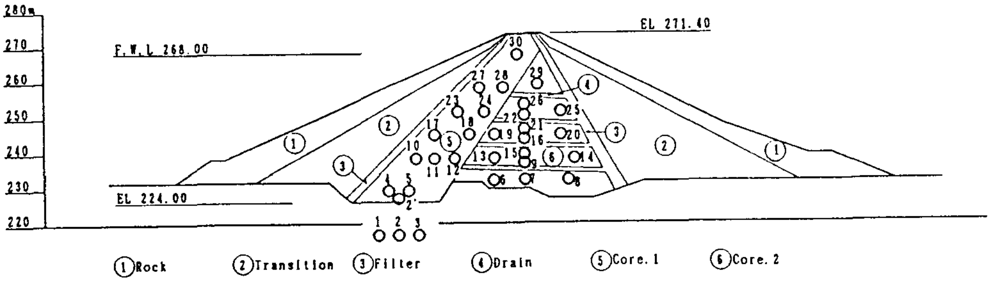
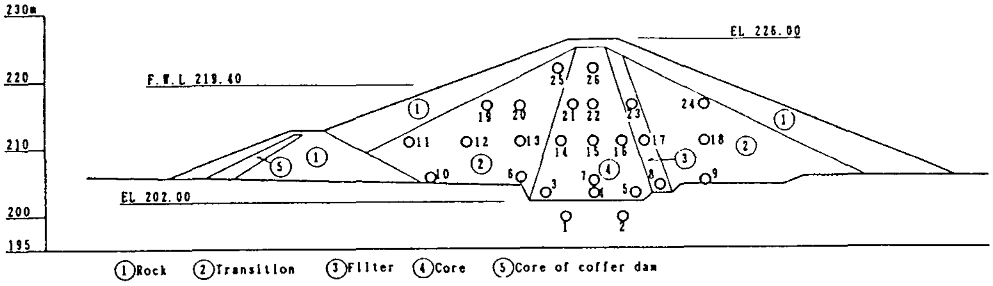


A Dam



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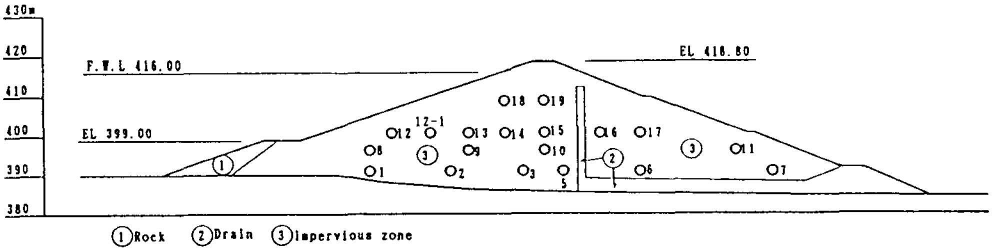
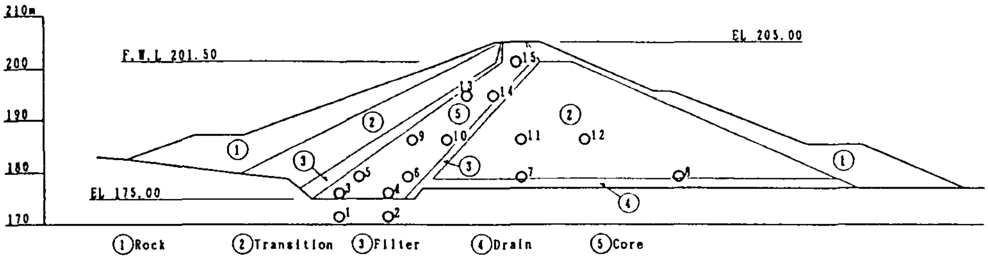
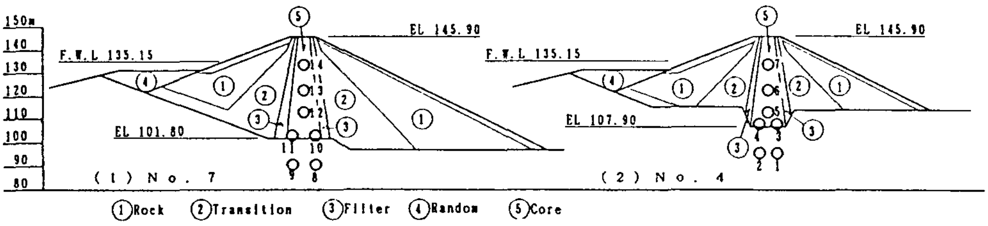


C Dam

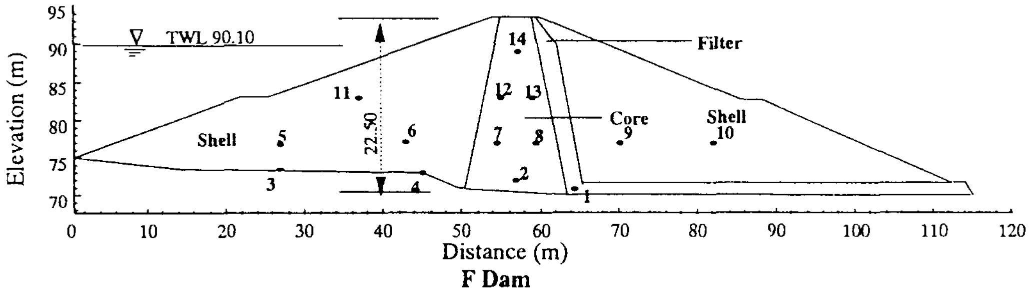


D Dam



E Dam

O Number and Location of Piezometer



F Dam

Figure 1 Representative Sections and Locations of Piezometers in A, B, C, D, E and F Dams

Table 1 Characteristics of Dams

	A Dam	B Dam	C Dam	D Dam	E Dam	F Dam
Type	I.C.R1)	C.C.Z1)	H.E1)	I.C.R1)	C.C.R1)	C.C.Z1)
Height(m)	47.4	24.0	33	30	43.6	22.5
Length(m)	192.0	137.0	165.0	197.0	208.0	136.2
Fill Volume (*10^3 m^3)	337.0	90.0	260.0	202.0	374.0	112.0
Reservoir Capacity (*10^3 m^3)	630.0	333.0	302.0	640.0	275.0	218.0
Piezometer Type	Calson	Calson	Twin-tube	Twin-tube	Calson	Twin-tube
No. of Piezometers	30	26	20	15	14	25
Completion of Dam	1972.12	1982.6	1981.12	1986.2	1988.12	1991.10
First Filling	1974.6	1983.7	1983.11	1987.2	1989.1	1992.5

1) **I.C.R.**: Inclined Core Rockfill Dam, **C.C.R.**: Central Core Rockfill Dam, **C.C.Z.**: Central Core Zoned Fill Dam and **H.E.**: Homogeneous Earth-fill Dam

governing equation of flow through the saturated and unsaturated porous media is basically derived from the Richard's equation of continuity and the Darcy's equation of motion. The equation of continuity for transient soil water movement in the form of partial differential equation can be expressed by the Richard's equation (1) as:

$$-\mathrm{div}\ v = -\nabla \cdot \rho v = \partial (r\theta)/\partial t \tag{1}$$

which is an expression of conservation of mass, where ρ is the density of water, v is the specific flux or Darcy velocity and θ is the volumetric moisture content, defined as the volume of water per unit volume of soil. Here, the density of water is assumed to be independent of space and time. Darcy's law to flow in porous media can be assumed applicable also to the unsaturated flow besides the saturated flow in the anisotropic media in the form:

$$v_i = -K_{ij}(\theta)\,\partial h/\partial x_i\ , \quad i,\ j = 1,2,3 \tag{2}$$

where $K_{ij}=K^s_{ij}K_r$, v_i is the volumetric flux (volume of water per unit area per unit time), $\partial h/\partial x_j$ is the hydraulic gradient, K_r is the relative permeability ($0 \leq K_r \leq 1$) and K^s_{ij} is the coefficient of permeability at saturation. K_r is the function of the volumetric water content (θ) or capillary pressure head ϕ, and is not a constant in the unsaturated zone. Employing the above two equations, the governing equation (3) of flow through the saturated and unsaturated porous media can be derived as follows:

$$\mathrm{div}\ K(\phi)\,\nabla\,(\phi + x_3) = \{C(\phi) + \alpha \cdot S_s\}\frac{\partial \phi}{\partial t} \tag{3}$$

where α=0 or 1 for unsaturated and saturated conditions respectively, ϕ, the pressure head, $K(\phi)$, the coefficients of permeability, x_3, the elevation head, $C(\phi)$, the specific moisture content, S_s, the specific storage and θ, the volumetric moisture content.

The above governing equation (3) of seepage flow where the pressure head, ϕ is to be determined at all nodes in the domain are subjected to boundary conditions as follows:

a) Initial Condition : $\phi(x_i, 0) = \phi_0(x_i)$ (4)

b) Prescribed Potential Boundary : $\phi(x_i, t) = \phi_t(x_i, t)$ (5)

c) Prescribed Flux Boundary : $\frac{\partial}{\partial x_i}(K_{ij}\frac{\partial \phi}{\partial x_j} + K_{i3})\,\overline{n} = -v(x_i,t)$ (6)

where $\overline{n}$ denotes the unit normal to the boundary and v is the velocity flux.

The unsaturated parameters of coefficients α ,n of the Van Genuchten's formula from which the negative capillary pressure, $\phi(\theta)$ and the relative coefficient of permeability, $K_r(\theta)$ are determined as given in equations (7) and (8) as follows:

$$|\phi| = \frac{1}{\alpha}(S_e^{-1/m} - 1)^{1/n} \ , \ \alpha > 1 \tag{7}$$

$$K_r = S_e^{1/2}\{1 - (1 - S_e^{1/m})^m\}^2 \tag{8}$$

where m=1/n, $S_e=(\theta-\theta_r)/(\theta_s-\theta_r)$; the effective degree of saturation, θ_r,θ_s; the minimum and maximum saturated volumetric moisture content respectively. With known ϕ, θ can be computed from equation (9) as follows:

$$\theta = \theta_r + (\theta_s - \theta_r)/(1 + |\alpha\phi|^n)^m \tag{9}$$

The continuous curve lines in Figure 6 shows the relationship of ϕ and K_r verses θ as described by equations (7) & (8) above.The required specific moisture content in the seepage analysis can also be computed from the equation (10) as shown below:

$$C(\phi) = \alpha(n-1)(\theta_s - \theta_r)S_e^{1/m}(1 - S_e^{1/m})^m \tag{10}$$

In the present analysis of seepage flow, S_S is neglected. Values of K_Y are estimated by the laboratory and field tests and as shown in Table 2, while the values for K_X are assumed equal to certain proportion of K_Y. The unsaturated coefficients of α and n in equations (7) and (8) are estimated based on the available laboratory measured data for various types of soils. The boundary conditions of reservoir water level fluctuation with time are based on the available daily records at the dam sites. In the analysis, only the impervious and/or semi-impervious zones are considered because the difference of the coefficients of permeability of the impervious and the pervious zones is more than 3000 times. The number of finite elements of the mesh made for each dam for the numerical analysis ranges from 115 for Dam D to 451 for Dam A.

2.2 Prediction of Pore Pressures by Back Analysis

By making use of the available monitored pore water pressure data in the early stage of reservoir fluctuation of first filling, back analysis was carried out to estimate the best fit seepage parameters assuming that the monitored readings are the true values. The back analysis of seepage flow involves the normal seepage analysis and the modification of the parameters concerned which are carried out by an iterative process until the best fit parameters are obtained. For the modification of the parameters, the Marquadt's method (for cases of A, B, C, D and E Dams) and the modified Gauss-Newton's method (for the case of F Dam only) based on minimizing the residue squares are employed. The objective function to be minimized is as follows:

$$S(x)=\sum_{i}^{n} w_i\{y_i-f_i(x)\}^2=\min. \tag{11}$$

where S(x), the sum of the residue squares, w_i , the weight factor, x, the parameters in the problem, y, the monitored readings of n number and f(x), the calculated readings.

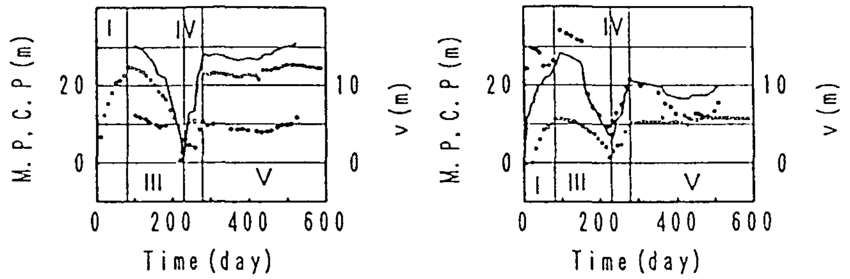
In the present back analysis, the five parameters considered for identification in the homogeneous impervious zone are K_x, K_y, S_s, α and n. The required input of these initial parameters are as shown in Table 2. All other conditions of back analyses are as defined previously for normal seepage analysis. The results of the estimated parameters are also shown in Table 2. Also shown in the table is K_y of each dam from laboratory tests for comparison.

The identified parameters are then used for further prediction of pore water pressures in subsequent stages of reservoir level fluctuations of up to the whole monitored durations. This is carried out using the normal seepage analysis. Except for using the identified parameters, all other conditions defined for the analysis remained the same.

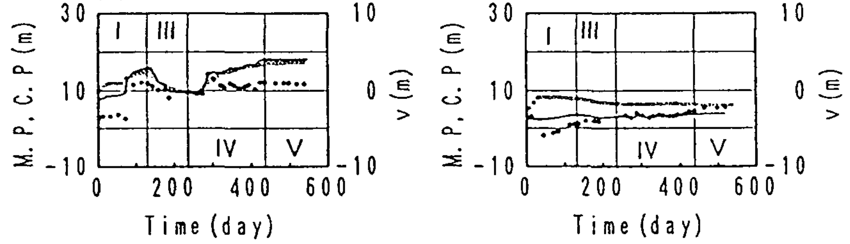
3. Results and Discussion on Normal Seepage Analysis

3.1 Change of Pore Pressure with Time

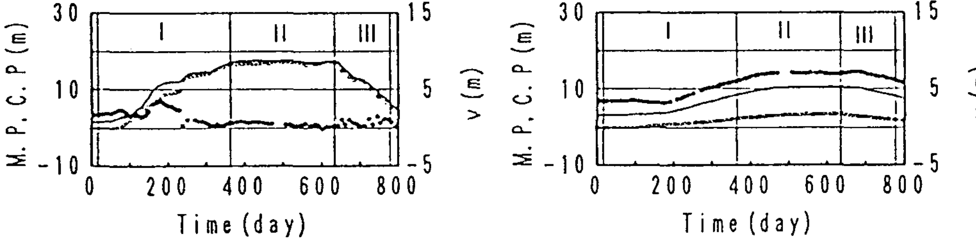
Fig. 2 shows the fluctuation of the calculated and the observed pore pressure head (total pressure head) with time for each dam. Also shown in the graphs are the ‘difference of prediction’, defined by the v=M.P.-C.P. The above plots of 2 representative monitored points for each dam are as shown, with each located at upstream and downstream of the impervious zone. As all the calculated pore water pressures are greater then 0 (m), all the monitored locations were at the saturated condition. The calculated pressures generally resemble the trend of the monitored pressures although the ‘difference of prediction’, v are larger at the downstream locations.



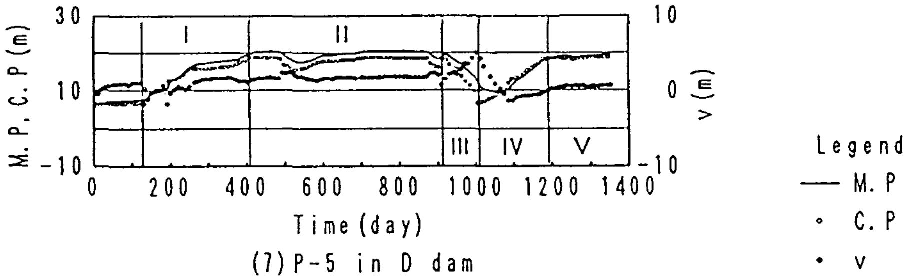
(1) P-10 in A dam (2) P-12 in A dam



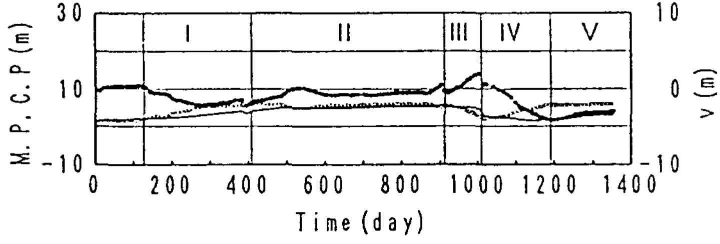
(3) P-3 in B dam (4) P-5 in B dam



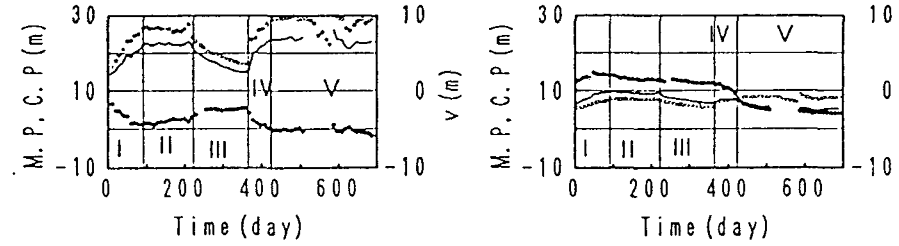
(5) P-8 in C dam (6) P-10 in C dam



(7) P-5 in D dam



(8) P-6 in D dam



(9) P-11 in E dam (10) P-10 in E dam

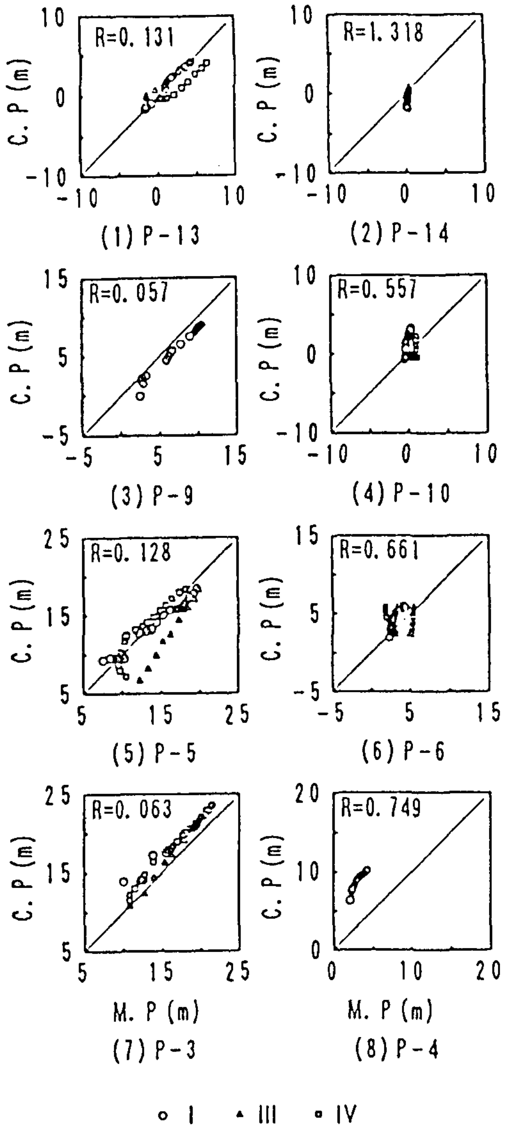
Fig. 2 Relationship of Monitored(M.P.), Calculated Pore Water Pressures(C.P.) and Difference (v=M.P-C.P) with Time

Table 2 Initial and Identified Best Fit Parameters of Impervious Zones from Back Analysis

Dam		K_x(cm/sec)	K_y(cm/sec)	S_s(1/cm)	α(1/cm)	n
A	**Initial**	$1.16*10^{-4}$	$1.16*10^{-7}$	$1.00*10^{-7}$	0.0100	1.4000
	Identified	$8.26*10^{-4}$	$1.45*10^{-8}$	$2.14*10^{-7}$	0.0949	1.4600
	Lab. Test	-	$1.66*10^{-5}$	-	-	-
B	**Initial**	$1.16*10^{-6}$	$1.16*10^{-6}$	$1.0*10^{-5}$	0.0100	1.8000
	Identified	$5.84*10^{-7}$	$2.05*10^{-6}$	$3.36*10^{-5}$	0.0070	4.6700
	Lab. Test	-	$3.10*10^{-6}$	-	-	-
C	**Initial**	$1.16*10^{-6}$	$1.16*10^{-6}$	$1.00*10^{-5}$	0.0100	1.400
	Identified	$4.41*10^{-6}$	$2.18*10^{-5}$	$2.86*10^{-5}$	0.0190	1.340
	Lab. Test	-	$7.00*10^{-7}$	-	-	-
D	**Initial**	$1.16*10^{-6}$	$1.16*10^{-6}$	$1.00*10^{-5}$	0.0010	1.800
	Identified	$5.00*10^{-6}$	$2.64*10^{-7}$	$3.13*10^{-5}$	0.0010	2.300
	Lab. Test	-	$1.74*10^{-5}$	-	-	-
E	**Initial**	$1.16*10^{-6}$	$1.16*10^{-6}$	$1.16*10^{-6}$	0.0100	1.8000
	Identified	$3.01*10^{-7}$	$5.29*10^{-13}$	$7.70*10^{-6}$	0.0010	1.8600
	Lab. Test	-	$2.04*10^{-5}$	-	-	-
F	**Initial**	$1.12*10^{-4}$	$5.10*10^{-5}$	$1.00*10^{-5}$	0.0101	2.137
	Identified	$1.14*10^{-4}$	$7.03*10^{-5}$	$1.00*10^{-5}$	0.0101	2.635
	Lab. Test	-	$1.00*10^{-6}$	-	-	-

3.2 Comparison of Calculated and Monitored Pore Pressures

Figures 3 and 4 shows the relationship of the monitored and calculated pore pressures in the impervious zones. The plots in Figure 3 actually shows the fitting goodness of prediction of the monitored pore pressures at the various monitored locations from low to high, upstream and downstream locations of the impervious zone for D Dam. Figure 4 shows the same plots of the representative monitored points at the lower regions of the upstream (left) and downstream (right) of the impervious zone of the other four dams. The diagonal line in each plot is the equivalent line whereby in the ideal case, when all the plotted points lie exactly along it, shows that the calculated values equal to the monitored values. As seen from Figure 3, most points at the upstream region are displaying good correlation and compatibility. However the plotted points of the downstreamlocations are clustered or deviating from the diagonal lines, showing no good correlation and compatibility. These same observations can also be seen from Figure 4 for the cases of A, B, C , E and F Dams. The plotted points for the cases of reservoir filling (symbols ○ & □) and falling (symbol △) generally coincide with one another except for the points located at the downstream of A and B Dams. It is thus assumed that the locations at downstream of A and B Dams are unsaturated zones.



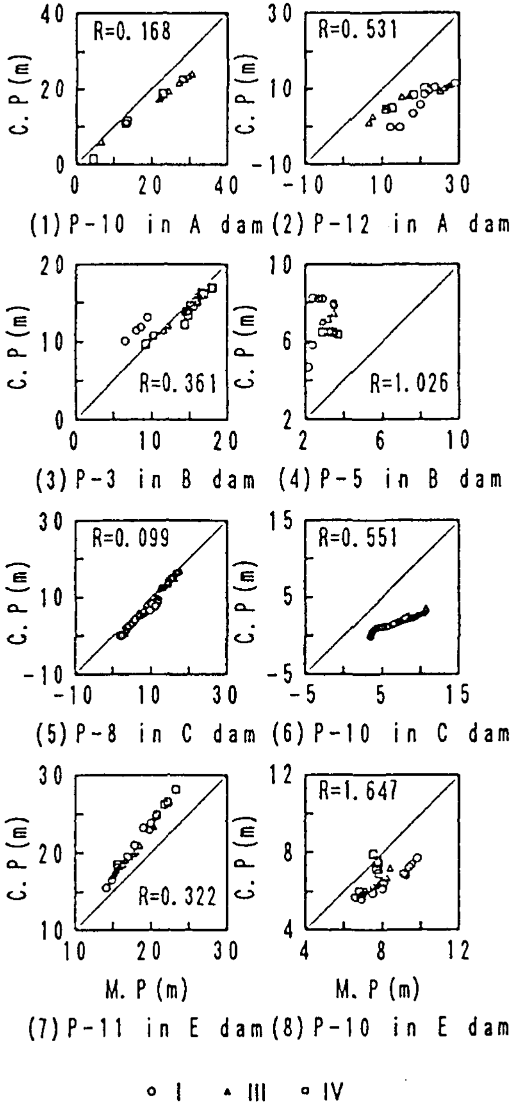
(1) P-13 (2) P-14

(3) P-9 (4) P-10

(5) P-5 (6) P-6

(7) P-3 (8) P-4

Fig. 3 Monitored(M.P.)-Calculated Pore Pressures(C.P.) for D Dam



(1) P-10 in A dam (2) P-12 in A dam

(3) P-3 in B dam (4) P-5 in B dam

(5) P-8 in C dam (6) P-10 in C dam

(7) P-11 in E dam (8) P-10 in E dam

Fig. 4 Monitored(M.P.)-Calculated Pore Pressures(C.P.) for A, B, C and E Dams

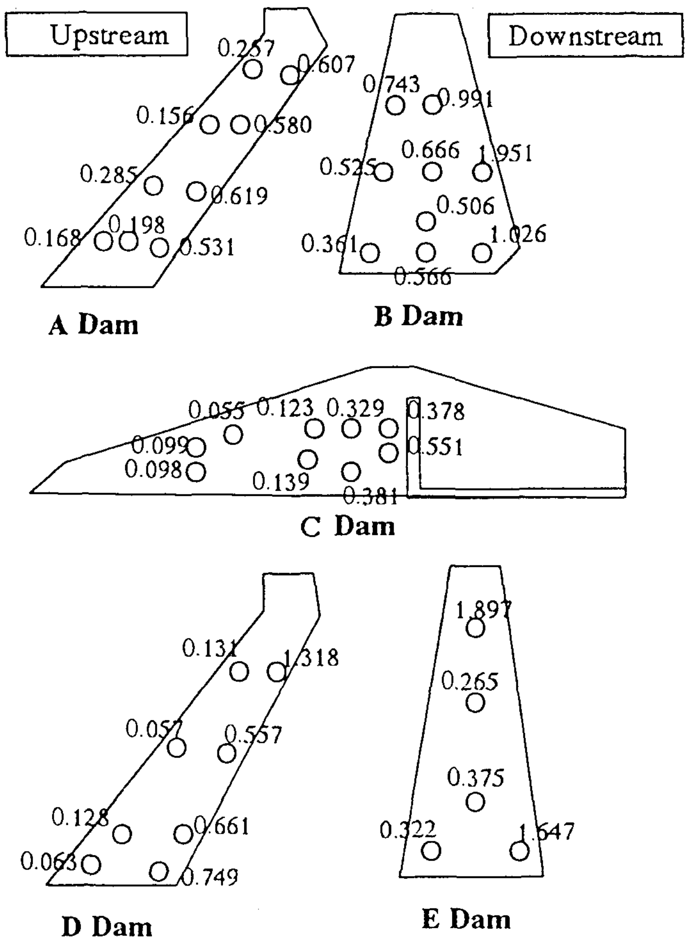


Fig. 5 Distribuion of Error Indices from Normal Seepage Analysis

3.4 Definition of Error Index and Its Distribution

Instead of evaluation by visual inspection of the plots as shown in Figures 4 and 5, quantitative evaluation will be defined in this section. The calculated values actually predict the monitored values satisfactorily if the correlation coefficient, r of the linear regression relationship between the calculated and monitored values is almost equal to unity. However, this alone does not necessarily reflect the compatibility of the calculated and monitored values such as for the case when the regressed line deviates from the equivalent diagonal line. Thus, the inclination of this line, a must also then be considered whereby a and r must be unity when the calculated and the monitored values coincide and correlate completely at the arbitrary reservoir water level or time. Thus, the error index, R, is defined by the equation (12) as:

$$\mathrm{R} = \sqrt{(r-1)^2 + (a-1)^2 + b^2} \quad , b=0 \tag{12}$$

where b is the intercept correlogram. Since b reflects the initial condition influenced by conditions such as methods of installation of the instruments or any other local conditions etc., the term b in the above equation is ignored. R is equal to zero for the ideal case when all the calculated and the monitored values are exactly equal. Thus, R serves as an indicator index to quantify the compatibility of the calculated and the monitored values.

In Figures 3 and 4, the error index of each plot is as indicated. Figure 5 shows the distribution of error indices of each dam. It can be seen that the accuracy of prediction of the pore pressures can easily be compared among the different points within the same dam and also among different dams. In general, the error indices at the downstream or lower region shows smaller values than the error indices of the upstream or upper region of the impervious core for each dam.

4. Results and Discussion of Back Analysis of Seepage

The identified best fit parameters by back analysis are as shown in Table 2. Figure 6 shows the unsaturated seepage characteristics based on the identified best fit parameters. Figure 7 shows the relationship of the monitored and calculated values at the representative observation points at the up-stream (left) and the downstream (right) of the impervious zone of each dam. Comparing to the error indices of the upstream locations obtained from the normal seepage analysis (R=0.018-0.168), the error indices computed using the best fit parameters from back analysis, has shown some improvement as indicated by the smaller R. As for Dam A (Figure 7(2)), the plotted points of the calculated and the monitored values for downstream location also lie close to the diagonal line showing a small R (0.194). However, for the other dams, R values are bigger at the downstream locations where R ranges from 0.413 to 2.155.

Figure 8 shows the distribution of the R obtained using the best fit parameters at the impervious zones for all the six dams. The error index value at the top (R_1) of each monitored point is the value obtained using the normal analysis up to the duration of first filling of the reservoir only, while the error index value at the bottom (R_a) of each point is the result of the normal analysis for

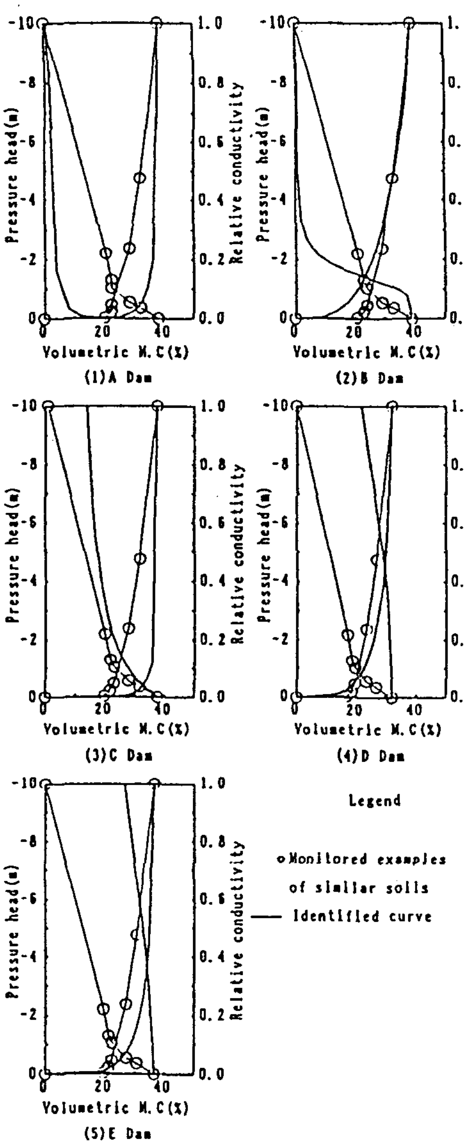


Fig. 6 Relationship of Pressure Head and Relative Conductivity - Volumetric Moisture Content

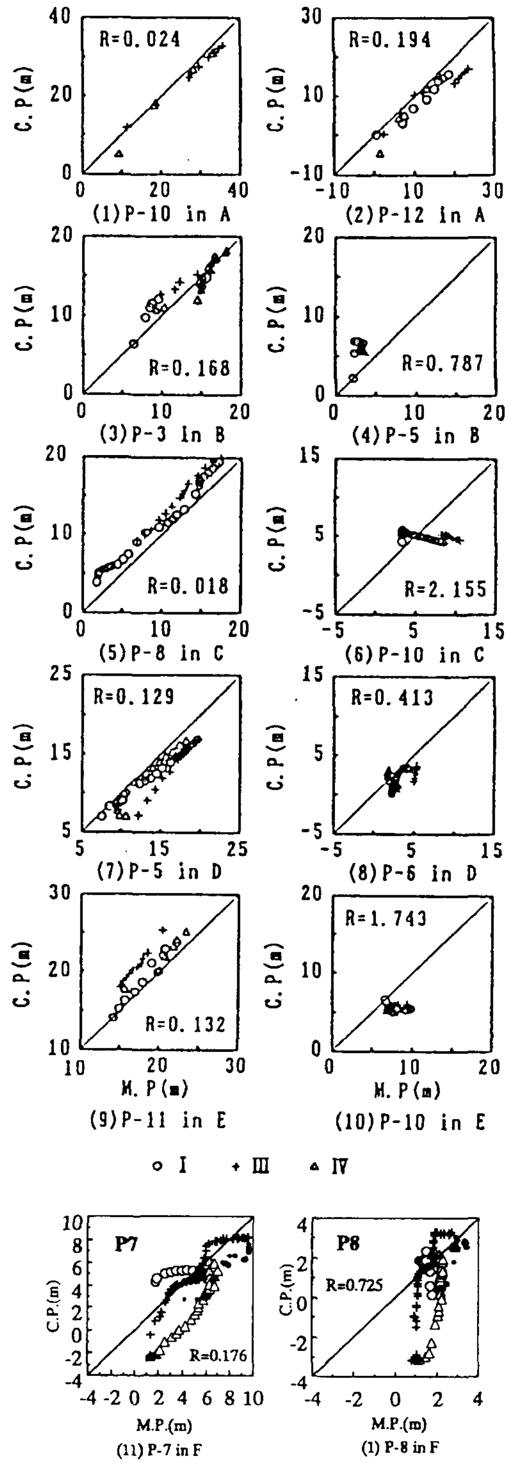
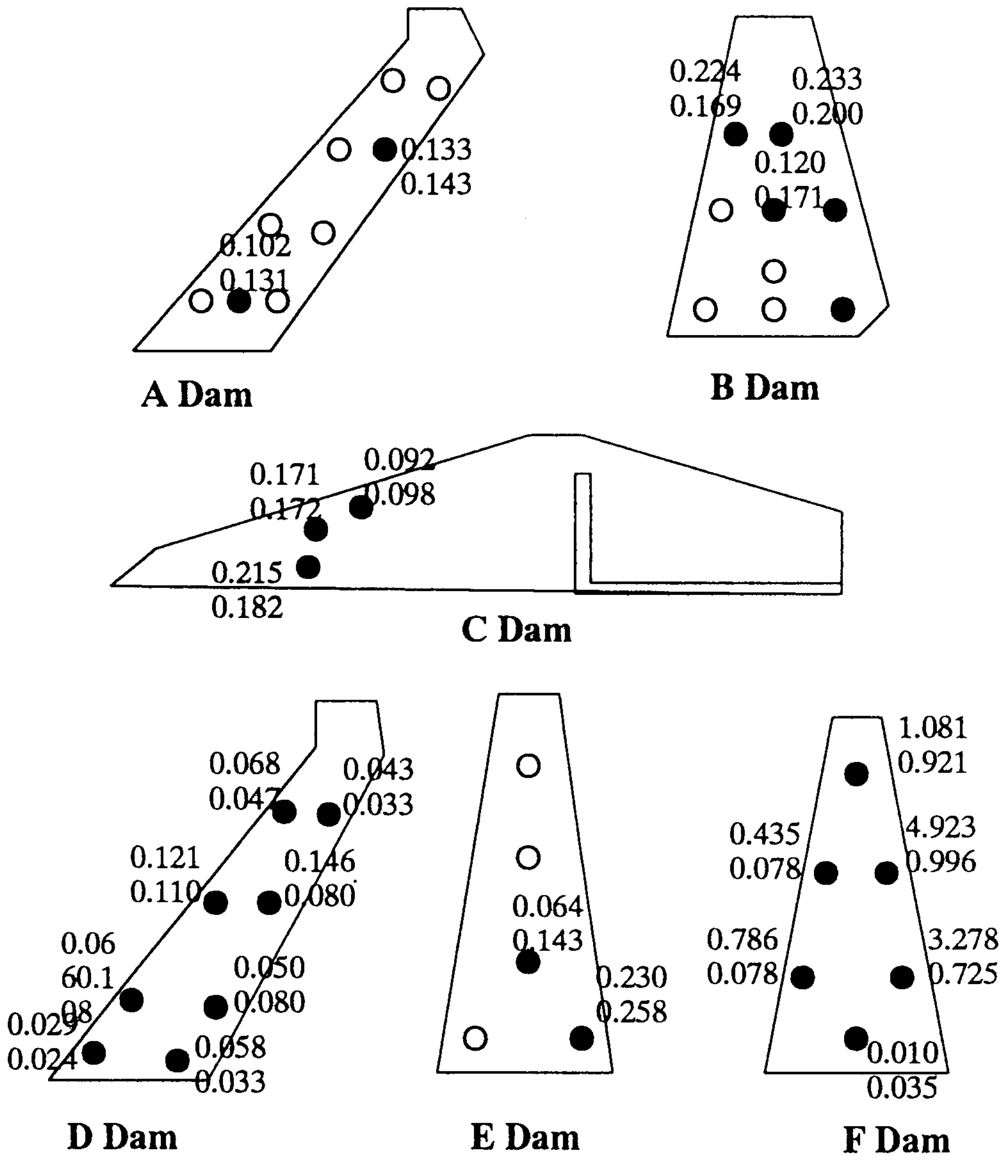


Fig. 7 Calculated - Monitored Pore Pressures from Back Analysis



Upper value , R1 of normal seepage analysis
up duration of first filling of Reservoir
Lower value, Ra of analysis up to the whole
monitored duration

Figure 8 Distribution of Error Indices using Estimated Parameters from Back Analysis

the whole duration of the monitored period. As seen from the figures, R_a is showing the same trend of consistency with R_l where the point with the smallest R_l is also the point with the smallest R_a. This means that the identified parameters up to duration of first filling of reservoir is also applicable for the whole monitored duration. Thus, the identified parameters with the monitored data in the first stage of filling of reservoir can possibly be used for further prediction of the pore water pressures of subsequent future monitored durations.

5. Conclusions

Saturated-unsaturated seepage analysis has been carried out on six types of fill dams located in western Japan. The prediction difference and the error index defined by the coefficients of correlation and regression, were used to make quantitative description of the compatibility between the calculated and the monitored pore water pressures. Back analysis using the monitored data to compute for the best fit parameters of the saturated-unsaturated seepage flow has also been carried out for each dam.

The main findings of the analysis are as follows:

(1) The error index can be used to describe quantitatively the compatibility between the calculated and the monitored pore water pressures.

(2) Based on the error index, comparisons of the calculated and the monitored values of the observation points within the same dam and also with other dams are possible.

(3) In any of the six dams, the error indices of observation points in the upstream sides of the impervious zones are smaller than those in the downstream side.

(4) The identified parameters with monitored data in the first stage of filling can be used for further prediction of pore water pressure with reasonable accuracy.

(5) Using this method, the reasons for the 'errors' of the monitored values can be deduced and easily estimated.

6. References

Y. Onishi & M. Nishigashi (1981). Menual for Saturated-unsaturated Seepage Analysis by FEM. *Kyoto University, Faculty of Engineering*

H.Fujii & T.Watanabe (1983). Pore Pressures of Small Fill Dams. *International Symposium on Field Measurements in Geomechanics, Zurich.*

R.A. Freeze (1971). Influence of Unsaturated Domain on Seepage Through Earth Dams. *Water Resour Res, Vol.7, No.4 , 929-941*

H. Fujii et al (1995) : Comparing Calculated and Monitored Pore Water Pressures of Five Fill Dams, *Unsaturated Soils (Edited by E.E. Alonzo P. Delage) pp273~290*

M.Nishigaki and Y.Takeshita(1993) : Research on Unsaturated Seepage Characteristics of Soils Based on Laboratory Test, *Faculty of Engineering, Okayama University*