Journal of the Faculty of Environmental Science and Technology, Okayama University
Vol.1, No.1, pp.193-198, March 1996

Settlement Prediction Method Using Observed Settlement Velocity

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(Received January 16 , 1996)

Abstract This paper presents a new method for prediction of consolidation settlements of soft grounds. The method is based on the theoretical result which shows that the settlement velocity of soft grounds non-improved or improved with sand drains decreases exponentially with time. Final settlements can be easily derived from the regression analysis for the relationship between the elapsed time and the observed settlement velocity. The method has advantages of its simplicity and capability to give the satisfactorily good estimate of the consolidation settlements, and also the support of the theoretical background.

Key Words : *consolidation, settlement prediction, soft ground, sand drains, one-dimensional consolidation*

1. INTRODUCTION

We sometimes fail to obtain the acceptable estimate of consolidation settlements of soft grounds even though adopting an elaborate constitutive equation in finite element method. The failure is probably due to uncertainty of soil constants and boundary conditions, etc. Accordingly, several methods to predict settlements using observed data have been proposed and widely used. (Yoshikuni et al. 1981)

This paper presents a new method of settlement prediction using observed settlement velocity. The method is based on the theoretical result which shows that the settlement velocity of soft grounds non-improved or improved with sand drains decreases exponentially with time. The final settlement can be easily derived from the regression analysis for the relationship between the elapsed time and the observed settlement velocity.

2. RELATIONSHIP BETWEEN TIME AND SETTLEMENT VELOCITY IN SOFT GROUND IMPROVED WITH SAND DRAINS

The approximate solution of Barron's consolidation equation for a sand drain (Barron 1948, Richart 1957, and Yoshikuni 1979) is as follows:

$$S(t) = S_f\{1 - \exp[\alpha t]\} \qquad (1)$$

where t is the time measured from the start of consolidation, $S(t)$ the settlement at the time t,

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S_f the final settlement, and

$$\alpha = \frac{-8}{F(n)} \frac{c_h}{d_e^2},$$

$$F(n) = \frac{n^2}{n^2-1}\ln(n) - \frac{3n^2-1}{4n^2}, \quad n = \frac{d_e}{d_w}$$

where c_h is the coefficient of consolidation, d_e is the equivalent effective diameter of a sand drain, and d_w the diameter of a sand drain.

The differentiation of Eq.(1) with respect to time (t) gives the settlement velocity (dS/dt),

$$\frac{dS}{dt} = -\alpha S_f \exp[\alpha t]. \tag{2}$$

Eq.(2) can be rewritten as follows:

$$y = A_0 \exp[A_1 x] \tag{3}$$

where $y = dS/dt$, $x = t$, and

$$A_0 = -\alpha S_f, \ A_1 = \alpha \tag{4}$$

Transforming the variables as $Y = \ln(y)$ and $X = x$, we obtain the following linear regression model.

$$Y = a_0 + a_1 X \tag{5}$$

where $a_0 = \ln A_0$ and $a_1 = A_1$.

Since settlement observations are usually made in a certain interval, we cannot obtain the derivative (dS/dt) like a mathematical derivative of a continuous function. Then we define the observed settlement velocity ($\Delta S/\Delta t$) instead of (dS/dt). Fig. 1 shows the calculation procedure of the observed settlement velocity. When we obtain observed data at Point A and B whose time and settlement are (t_a, S_a), (t_b, S_b) respectively, we define the observed settlement velocity at time t_c with the next equations:

$$\Delta S / \Delta t = \frac{S_b - S_a}{t_b - t_a} \quad \text{at } t_c = \frac{t_a + t_b}{2} \tag{6}$$

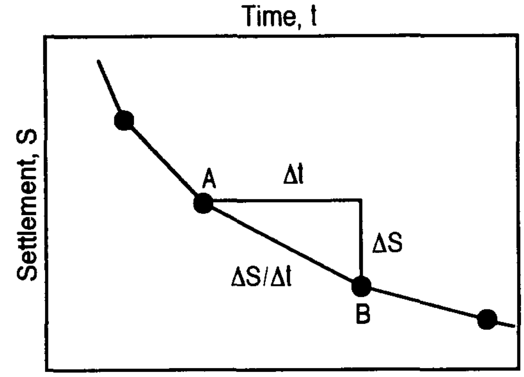

Fig. 1 Calculation of observed settlement velocity (ΔS/Δt)

Applying the model to the relationship between the elapsed time (t) and the observed settlement velocity ($\Delta S/\Delta t$), we can find the regression parameters a_0, a_1. Then we can calculate A_0, A_1 and the final settlement S_f from Eq.(4) as

$$S_f = -\frac{A_0}{A_1}, \tag{7}$$

and also the settlement curve from Eq.(1).

The settlement velocity during embankment should not be used in the analysis because Eq.(1) can be held only with such an assumption that all loads are applied simultaneously. Hence it is necessary to introduce a new coordinate system (t', S') whose origin is located at ($t = t_0$, $S = S_0$) on the settlement curve Eq.(1). We then obtain the following equation.

$$S'(t') = S'_f \{1 - \exp[\alpha t']\} \tag{8}$$

where $S(t) = S_0 + S'(t')$ and $S_f = S_0 + S'_f$.

Applying the same procedure to the relationship between the elapsed time (t') and the observed settlement velocity ($\Delta S'/\Delta t'$), we can obtain the final settlement S'_f from the regression parameters A'_0, A'_1 on Eq.(7) as

$$S'_f = -\frac{A'_0}{A'_1}, \tag{9}$$

and also the total settlement S as

$$S = S_0 + S'_f\{1 - \exp[\alpha t']\}. \quad (10)$$

3. DIFFERENCE BETWEEN ANALYTICAL AND APPROXIMATE SOLUTIONS OF BARRON'S CONSOLIDATION EQUATION

The relationships between the time factor (T_h) and the settlement velocity (dU/dT_h) for the different values of the diameter ratio (n) are shown in Fig. 2. The solid lines represent analytical solutions and the open circles the approximate solutions of Barron's consolidation equation. We notice that there are slight differences between the approximate solutions and the analytical solutions in the region of small T_h.

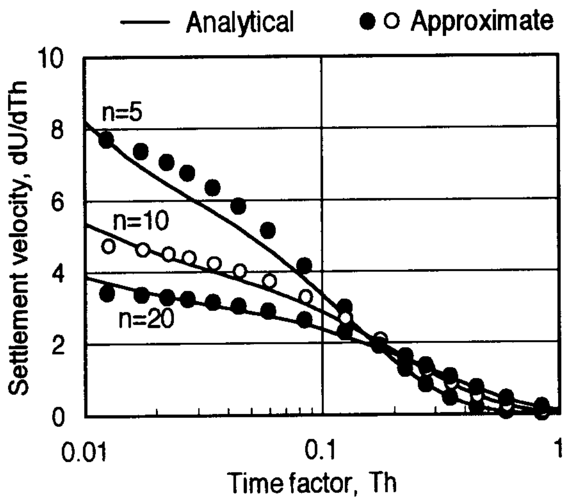

Fig. 2 Analytical and approximate relationships between time factor (Th) and settlement velocity (dU/dTh)

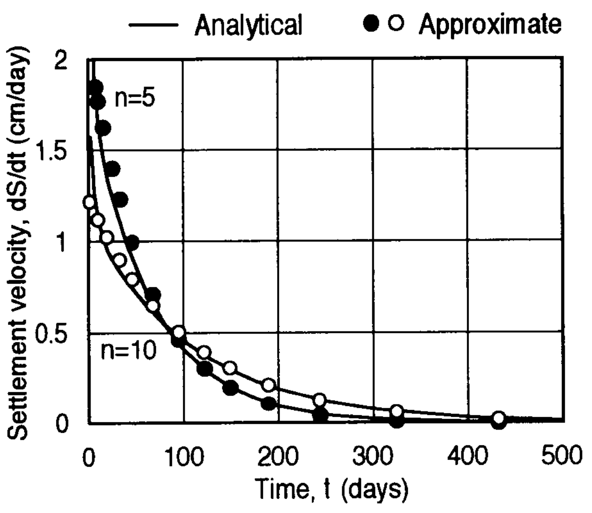

Fig. 3 Analytical and approximate relationships between time (t) and settlement velocity (dS/dt)

Fig. 3 shows the relationship between the real time (t) and the real settlement velocity (dS/dt) calculated with the soil constants of KASAOKA-Bay clay (Fujii et al. 1991). The differences appear only in the initial stage of consolidation. Those differences are therefore practically negligible.

4. RELATIONSHIP BETWEEN TIME AND SETTLEMENT VELOCITY IN NON-IMPROVED SOFT GROUND

Asaoka (1978) shows that the following equation is identical to Mikasa's consolidation equation for non-improved soft grounds.

$$S + c_1\dot{S} + c_2\ddot{S} + \dots + c_n S^{(n)} + \dots = S_f \quad (11)$$

where c_1, c_2, ... , c_n are constants, and dots refer to the differentiation with respect to time.

He also proposed a graphical settlement prediction method based on the following equation which is the first order approximation of Eq.(11).

$$S + c_1\dot{S} = S_f \quad (12)$$

The differential equation (12) can be easily solved as follows:

$$S(t) = S_f\{1 - \exp[-\frac{t}{c_1}]\} \quad (13)$$

Eq.(13) takes the same form as that of Eq.(1) for the settlement of the soft grounds improved with sand drains. The method previously proposed for improved soft grounds is therefore applicable to the settlement prediction of non-improved soft grounds.

Atkinson and Bransby (1978) obtained an approximate solution of Terzaghi's consolidation equation for non-improved soft grounds. They approximate the isochrones with parabolas. The solution has the same form as that of Eq.(13), and accordingly the settlement velocity decreases exponentially with time.

5. PREDICTED SETTLEMENTS OF SOFT GROUNDS IMPROVED WITH PACKED SAND DRAINS

The proposed method is applied to the settlement prediction of the soft grounds in KASAOKA-Bay reclaimed land, OKAYAMA. The ground had been improved with packed sand drains and preloading embankments (Fujii et al. 1991, Shimada et al. 1992).

Figs. 4 and 5 show the relationships between time (t') and the settlement velocity ($\Delta S'/\Delta t'$) for Point H in the field No.4 and No.7, respectively. The time of the completion of embankment is adopted as the origin of (t', S') coordinate system. The open circles show the observed settlement velocities. The exponentially decaying lines are the result of the regression analysis with the least square method. The

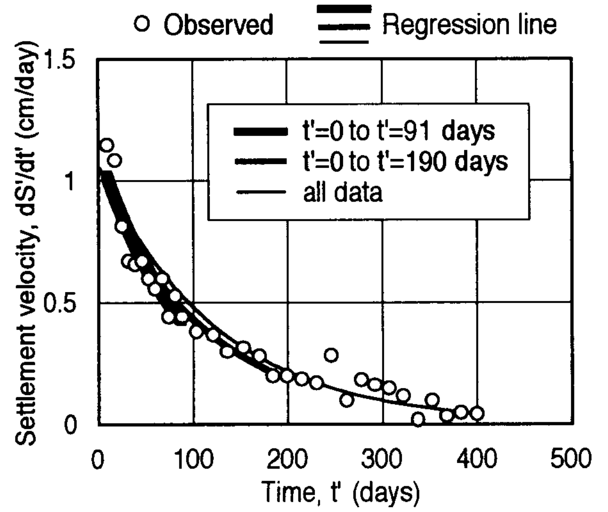

Fig. 4 Relationship between time (t') and settlement velocity (dS'/dt') of Point H in field No.4

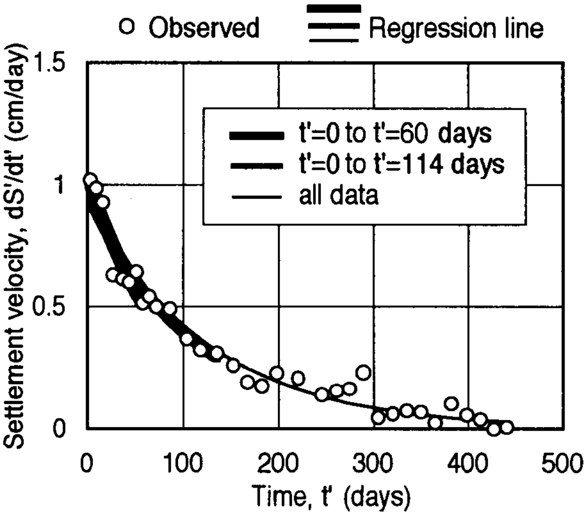

Fig. 5 Relationship between time (t') and settlement velocity (dS'/dt') of Point H in field No.7

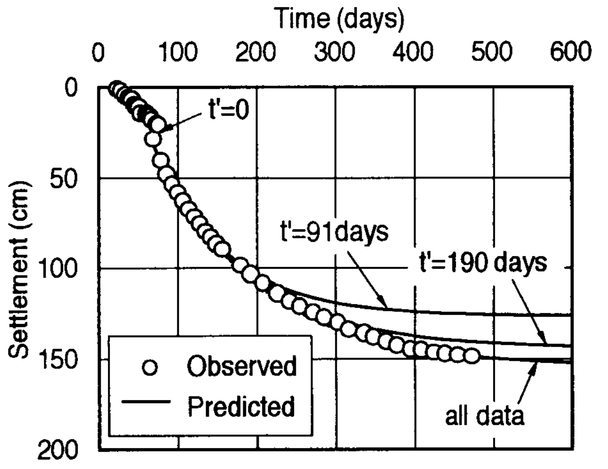

Fig. 6 Settlement prediction for Point H in field No.4

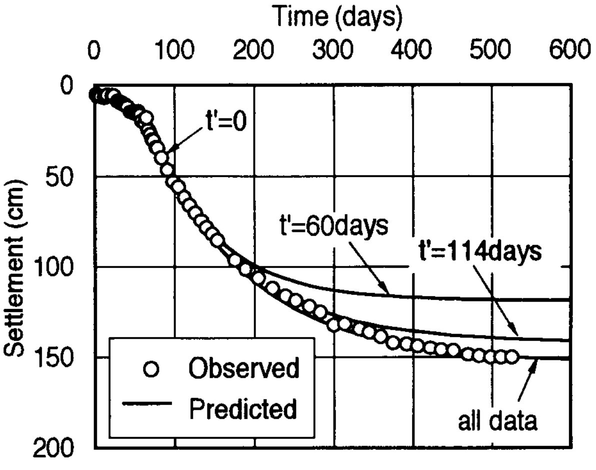

Fig. 7 Settlement prediction for Point H in field No.7

thin solid lines are derived from all observed data. The thick solid lines are derived from the data observed from $t'=0$ to $t'=91$ days, 190 days in Fig. 4, and $t'=0$ to $t'=60$ days, 114 days in Fig. 5, respectively.

Fig. 6 shows the results of the settlement prediction for Point H in the field No.4. The open circles represent observed settlements. Three solid lines are settlement curves predicted at $t'=91$ days, 190 days and predicted with all observed data, respectively. The prediction is also applied to the Point H in the field No.7 and results are shown in Fig. 7. Three settlement curves are predicted at $t'=60$ days, 144 days and predicted with all observed data, respectively.

These figures show that the observed settlements can be expressed successfully when all observed data are used, i.e., the settlement of soft grounds improved with sand drains can be calculated with Eq.(1). It is also clear from the figures that the prediction accuracy becomes improved more with the longer elapsed time from the completion of embankment.

The prediction accuracy is presented in Fig. 8 as the relationship between U' and R_s. These parameters are defined with the following equations by Yoshikuni et al. (1981).

$$U' = \frac{\text{Observed settlement at the time of prediction}}{\text{Last observed settlement}}$$

$$R_s = \frac{\text{Predicted settlement for the time of last observation}}{\text{Last observed settlement}} \tag{14}$$

where U' refers to the equivalent degree of consolidation at the time of prediction. R_s shows the accuracy of prediction. When there is no error in prediction, R_s becomes unity.

Yoshikuni et al. (1981) compared the accuracies of several prediction methods with their observed data. Their results are summarized in Table 1 which shows U' at some levels of R_s. The smaller value of U' at the same level of R_s indicates good prediction.

The result of the proposed method with the data of KASAOKA-Bay reclaimed land is also presented in the table. The method gives the satisfactorily good estimate of settlements.

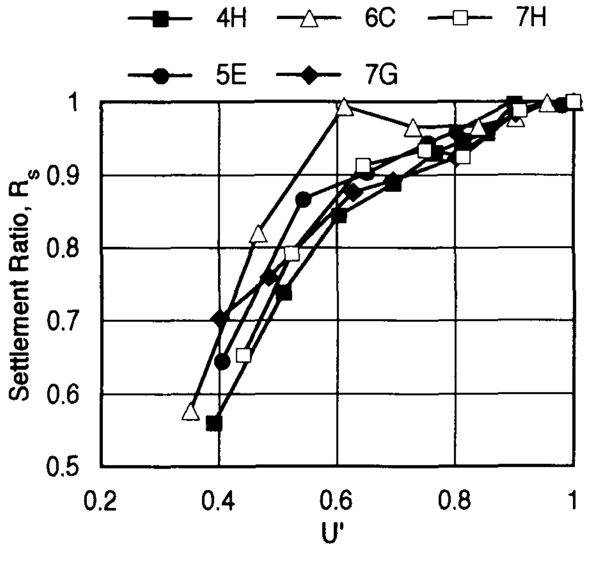

Fig. 8 Predicting accuracy in Kasaoka-bay reclaimed land

Table 1 Prediction methods and their accuracies (From Yoshikuni et al.(1981))

R_s Methods	R_s=0.7	R_s=0.8	R_s=0.9	R_s=0.95
Hyperbolic	U'=0.4	U'=0.6	U'=0.7	U'=0.8
Monden	U'=0.6	U'=0.7	U'=0.8	U'=0.9
Hoshino	U'=0.4	U'=0.5	U'=0.6	U'=0.7
Asaoka	U'=0.5	U'=0.65	U'=0.8	U'=0.9
Proposed	U'=0.5	U'=0.55	U'=0.7	U'=0.8

6. PREDICTED SETTLEMENTS OF NON-IMPROVED SOFT GROUND

Fig. 9 shows the relationships between time (t') and the settlement velocity ($\Delta S'/\Delta t'$) for a non-improved soft ground. The observed data is published by Aboshi (1969) and cited by Asaoka (1978). The time of the completion of embankment is adopted as the origin of (t', S') coordinate system. The open circles show the observed settlement velocities. The exponentially decaying lines are the result of the regression analysis with the least square method. The thin solid line is derived from all observed data. The thick solid line is derived from the data observed from $t' = 0$ to $t' = 550$ days.

Fig. 10 shows the results of the settlement prediction. The open circles represent observed settlements. Two solid lines are the settlement curves predicted at $t' = 550$ days and predicted with all observed data, respectively. The time of the completion of embankment is shown as $t' = 0$ in the figure. The proposed method has demonstrated its capability to predict the settlements satisfactorily.

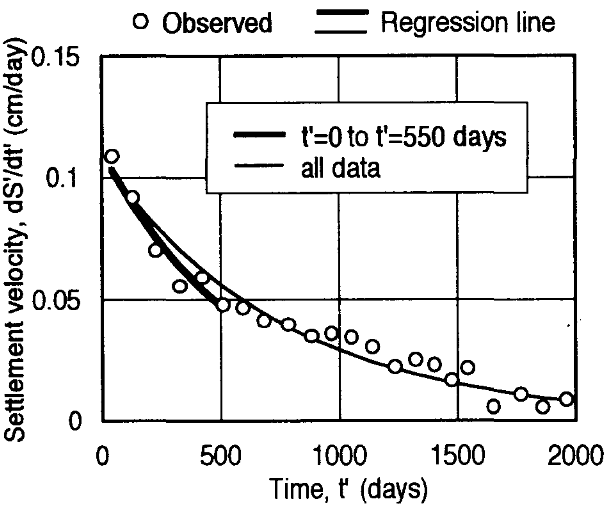

Fig. 9 Relationship between time (t') and settlement velocity (dS'/dt') of non-improved soft ground

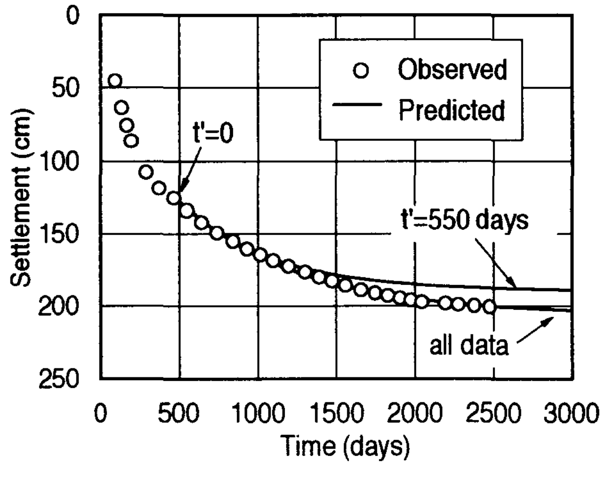

Fig. 10 Settlement prediction for non-improved soft ground

7. CONCLUSIONS

A settlement prediction method using the settlement velocity is proposed for soft grounds non-improved and improved with sand drains. The method has the advantages of its simplicity and capability to give satisfactorily good estimates of the consolidation settlements, and also the support of the theoretical background.

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