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Determination of Unsaturated Soil Hydraulic Properties by Transient Flow Experiments and Parameter Estimation

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SYNOPSIS

The numerical feasibility of determining soil water retention and hydraulic conductivity functions simultaneously from one-dimensional transient flow experiments in the laboratory by parameter estimation method is evaluated. Soil hydraulic properties are assumed to be represented by van Genuchten's closed-form expressions involving two unknown parameters: coefficients α and n . These parameters are evaluated by nonlinear least-squares fitting of predicted and observed pressure head with time. Gravity drainage experiments are performed for Toyoura standard sand to evaluate the adequacy of this proposed method.

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1. INTRODUCTION

Knowledge of the unsaturated soil hydraulic properties is essential requirement for prediction of seepage flow and contaminant transport through the vadose zone. The unsaturated soil hydraulic properties consist of the hydraulic conductivity as a function of pressure head and the soil water retention curve. pF tests are usually performed for determining the soil water retention curve in the laboratory. In pF tests, the soil column method is one-dimensional, vertical equilibrium desorption or absorption experiments. This method is widely used because its procedure and apparatus are very simple.[1]

There are, however, some disadvantages in this method as, 1) time-consuming data collection, 2) difficult to determine the equilibrium state, and 3) difficult to calculate the hydraulic conductivity. We are, therefore, apt to terminate measurements before experiments have reached to the equilibrium state and obtain an mistaken soil water retention information.

In this paper, a new soil column method is investigated. This method is gravity drainage experiments involving initially saturated columns, and pressure head data are measured to judge the equilibrium state of the soil columns. The numerical feasibility of estimating soil hydraulic properties simultaneously from the soil column method by a parameter estimation method is evaluated. Soil hydraulic properties are assumed to be represented by van Genuchten's closed-form expressions involving five unknown parameters. Three of the five model parameters are independently measured from laboratory tests and the other two are estimated by nonlinear least-squares fitting of predicted to observed pressure head with time. Our proposed soil column method are performed for Toyoura standard sand to evaluate limitations of this parameter estimation method imposed by constraints of uniqueness and sensitivity to error.

2. IMPROVED SOIL COLUMN METHOD

The soil column method which we perform is gravity drainage experiments involving initially saturated columns of fine sand (Fig.1). Pressure head data are measured with time by pressure transducers which are installed on the soil columns. If measured pressure head data are equal to the potential head at the measured points, it is judged that the phenomenon of the gravity drainage of soil columns has reached to the equilibrium state. The distribution of pressure head with time in the soil columns are calculated by the interpolation or the extrapolation of measured pressure head data at each point.

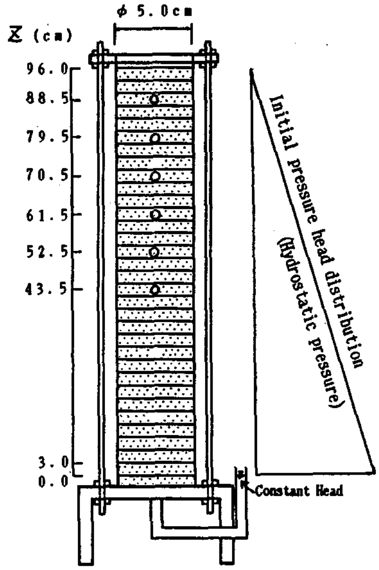


Fig.1 Gravity drainage experiments in the soil column method. (○: Pressure transducers)

3. DETERMINATION OF UNSATURATED SOIL HYDRAULIC PROPERTIES

3.1 Formulation of the Direct Problem

In this study we consider one-dimensional, vertical transient flow in a rigid porous medium governed by

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left\{k(h)\left(\frac{\partial h}{\partial z} - 1\right)\right\} \quad (1)$$

where h is pressure head, C is the soil water capacity, being the slope of the soil water retention curve (= $d\theta/dh$, θ is the volumetric water content), k is the hydraulic conductivity, t is time, and z is vertical distance taken positive upward.

The appropriate initial and boundary conditions are

$$dh/dx = 1 \qquad t = 0,\ 0 < z < L \quad (2a)$$

$$q = 0 \qquad t > 0,\ z = L \quad (2b)$$

$$h = h_a \qquad t > 0,\ z = 0 \quad (2c)$$

where z=0 is taken at the bottom of the soil columns, z=L is at the top of the soil columns, q is the outflow or inflow at the top of soil columns, and h_a is the atmospheric pressure.

The solution of Eq.(1) and (2) was obtained by finite element analysis of nonsteady unsaturated seepage flow.

3.2 Parametric Model for the Unsaturated Soil Hydraulic Properties

The unsaturated soil hydraulic properties are strongly nonlinear functions of the pressure head. It is assumed that suitable analytical expressions for these functions are available. We assume soil hydraulic properties are described by van Genuchten's closed-form expressions.[2]

$$Se = \frac{\theta - \theta r}{\theta s - \theta r} = \{ 1 + | \alpha \psi |^{n} \}^{-m} \quad (3)$$

$$k(h) = ks \cdot Se^{1/2} \{ 1 - (1 - Se^{1/m})^{m} \}^{2} \quad (4)$$

$$C(h) = \alpha (n-1)(\theta s - \theta r) Se^{1/m} (1 - Se^{1/m})^{m} \quad (5)$$

where m=1-1/n, Se is the effective saturation, θs is the saturated water content, θr is the residual water content, ks is the saturated conductivity, and α, n are the soil retention curve shape parameters (empirical parameters).

Expressions for k(h) and C(h) follow from Eq.(3) through Eq.(5). Of the five parameters ks, θs, θr, α, and n in these expressions, the first two have clear physical significance and are independently measured from laboratory tests. The residual water content is defined nominally as the water content at which k -> 0 and h -> $-\infty$. Literally, it is considered that the residual water content for sandy soils is equal to 0.0.

Respectively, the parameters α and n are inversely related to the air-entry value and width of the pore distribution. From our own data, α generally ranges from 0.02 to 0.1 [1/cm], while n usually varies from 3 to 10 for sandy soils.

In the parameter estimation problem we assume that ks and θs have been measured independently and θr is set equal to 0.0.Values of α and n are sought by the numerical inversion of one-dimensional unsaturated flow problem.

3.3 Parameter Estimation Procedure

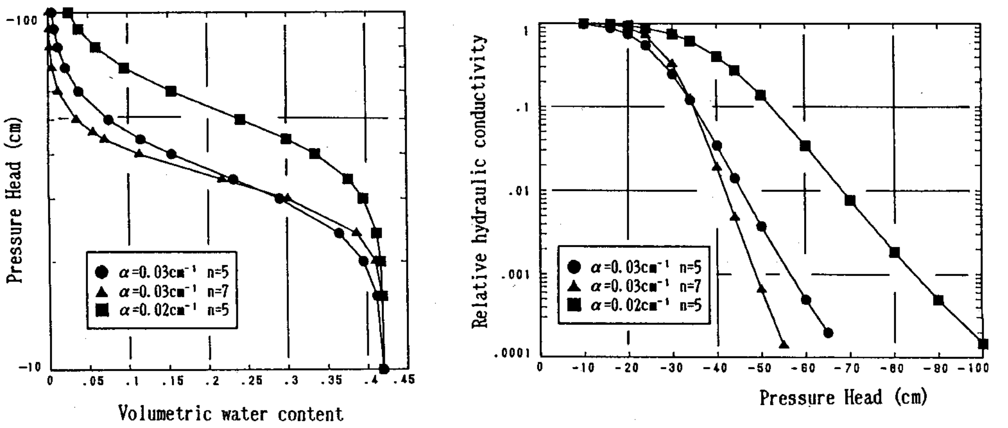
A set of pressure head measurements h at specific times t_i (i=1,2,...,N) are obtained from the soil column method results. These $h(t_i)$ are employed as input data for the numerical inversion problem. Let $h(\mathbf{b},t_i)$ be the numerically calculated values of pressure head corresponding to a trial vector of parameter values [**b**] where [**b**] is the two-dimensional vector [α , n]. The problem we pose is to find an optimal combination of parameters [**b'**] that minimizes the objective function:

$$E(b) = \sum_{i=1}^{N} \{ wi [h(ti) - h'(ti,b)] \}^2 \quad (6)$$

where Wi is a weighting function. we consider that Wi takes 1.0 here.

To determine [**b'**] we use an optimization algorithm based on the Levenberg-Marquardt method. This method represents an optimal combination of the method of steepest descent and the Gauss-Newton method, and widely used for nonlinear least-squares optimization.

We wish to investigate the adequacy of pressure head observed at times $t_1,\ldots,t_N$ to define unique solutions to the inverse problem. It is anticipated that the probability of nonunqeness will increase as $h(t_N)$ diminishes relative to the equilibrium pressure head $h(t_\infty)$, where t_∞ is the time to effectively reach to the equilibrium state. Fig.2 illustrates how the van Genuchten's model can describe different shape of soil water retention and relative conductivity curves by three combinations of the parameter values α and n. Parameter sensitivity analysis to the pressure head with time is performed by using these parameter combinations. To illustrate results, pressure head distributions during the gravity drainage in soil columns are plotted in Fig.3. This figure shows that sensitivities of pressure head are found, but the sensitivity to parameter n is low compared to parameter α .



(a) Soil water retention curve (b) Relative hydraulic conductivity curve

Fig. 2 van Genuchten's model for various parameter values of α and n.

4. APPLICATIONS AND DISCUSSION

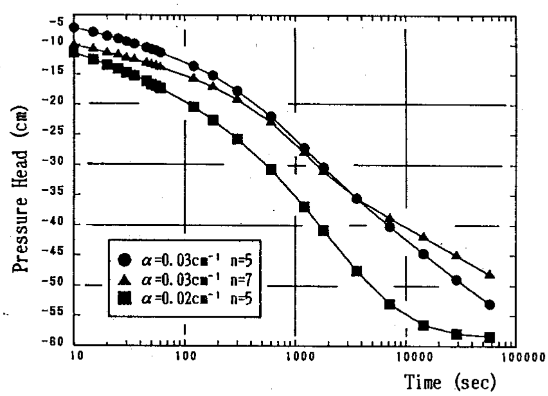


Fig. 3 Parameter sensitivity of pressure head with time to α and n.

Our proposed procedure was applied to the soil column method for Toyoura standard sand. The material was carefully packed into the acrylic columns 5-cm diameter, and 96-cm length with a particular dry density (γ_d=1.52 g/cm^3), saturated and then subjected to gravity drainage. The parameter ks and θs were independently measured, ks is 2.56x10^{-2} cm/s , and θs is 0.42. In this experiments, pressure head was measured at the same time for six different points in the soil columns by using pressure transducers (the range of measuring from -0.3 kgf/cm^2 to +0.3 kgf/cm^2, hysteresis is ±0.02% Fs) with -2kgf/cm^2 air-entry ceramic caps. The experiments were continued for about one week.

Measured pressure head with time is given in Fig.4 and pressure head distribution in the soil columns with time are shown in Fig.5. From these experiments results, pressure head measured from No.4 (z=61.5cm) to No.6 (z=43.5cm) has reached to the equilibrium state for about 6 days. We employ transient data for the first 900 sec of pressure head at No.5 (z=52.5cm) to the perform parameter estimation procedure.

Predicted soil water retention curve was compared with observed data from the experiments. As seen in Fig.6, a reasonable correspondence is obtained between estimated data and measured one.

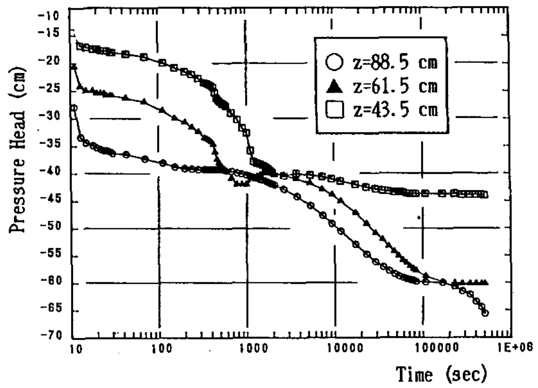


Fig. 4 Measured pressure head with time in soil columns.

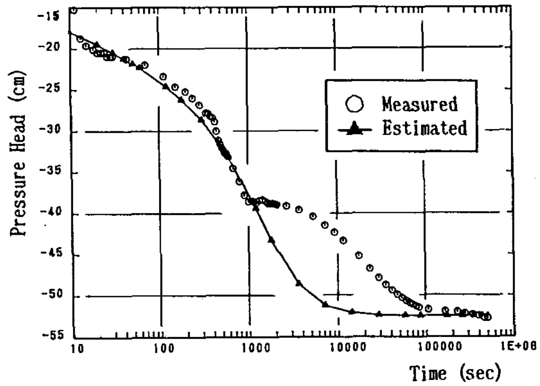


Fig. 5 Observed and estimated pressure head with time at z=52.5cm.

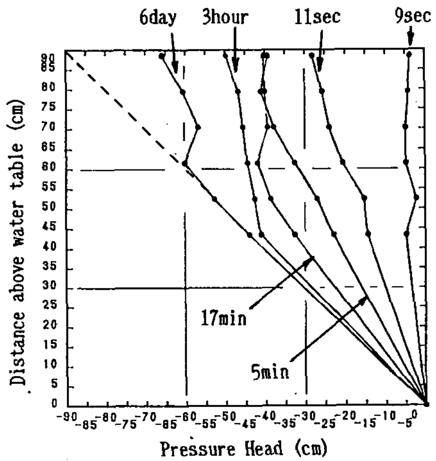


Fig. 6 Pressure head distribution with time in soil columns.

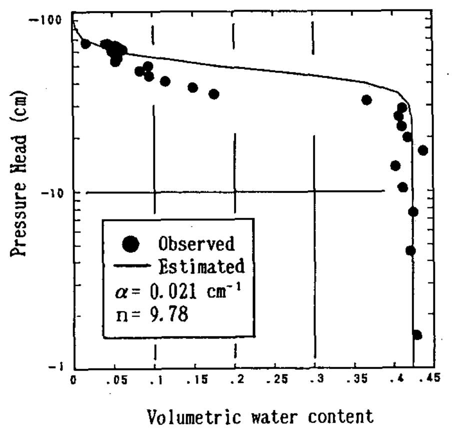


Fig. 7 Observed and estimated soil water retention curve for Toyoura standard sand.

5. CONCLUSIONS

An analysis of the inverse problem have been presented for the determination of unsaturated soil hydraulic properties by the soil column method. The conclusions obtained in this paper are as follows:

(1) The soil column method which we suggest is gravity drainage experiments involving initially saturated columns, and pressure head data in the soil columns are measured to judge the equilibrium state of the gravity drainage. This procedure is applied to Toyoura standard sand.

(2) A new method of estimating unsaturated soil hydraulic properties form the soil column method is proposed. In this method a nonlinear least squares algorithm incorporating finite element analysis of one-dimensional nonsteady unsaturated seepage flow is used. The advantages of the proposed method are in the possibility of identifying the optimal unsaturated soil hydraulic properties and diminishing experimental time.

(3) Soil hydraulic properties are assumed to be represented by van Genuchten's closed-form expressions involving five unknown parameters. These unknown parameters allow great flexibility in the shape of the hydraulic functions. Therefore when these functions are determined, the cumbersome handling of unsaturated soil hydraulic properties data in models of unsaturated flow can be avoided.

REFERENCES

[1] JSSMFE(1979):"Manual of Soil Laboratory Testing (2nd Edition)", Chapter 7. (In Japanese)

[2] van Genuchten,M.Th.(1980):"A closed-form equation for predicting the hydraulic conductivity of unsaturated soils.",Soil Sci.Am.J., Vol.144,pp.892-898.