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Characteristics of Errors in Open and Closed Trilateration Nets

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SYNOPSIS

Distance measurements have been more and more easy and accurate to carry out, and it is expected that distance mesurements may provide rather accurate results than angle measurements. Under these circumstances, caracteritics of errors in typical trilateration nets are investigated. The nets investigated are as follows: From single row of chains to pranimetrically extended nets in figure, open and closed networks with respect to external constraint, and with and without as to internal constraint. Computations are performed by use of the method of condition equations, and behaviours of error propagation and errors of coordinates of stations in the nets are shown in case of typical nets. For example, effects for decrease in error by composing a double row of chains and by enforcing external constraints are explained.

1. INTRODUCTION

In civil engineering survey works, surrounding field circumstances

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and requested accuracies differ from job to job, and it is now possible to adopt various methods and instruments to a control survey for such a engineering works. Then, profound and integrated considerations are necessary to select a suitable method for a job in hand(1~5). Electoromagnetic distance meters have been improved in this decade and it is expected that new instruments are useful for small or middle scale control surveying(1,6~8).

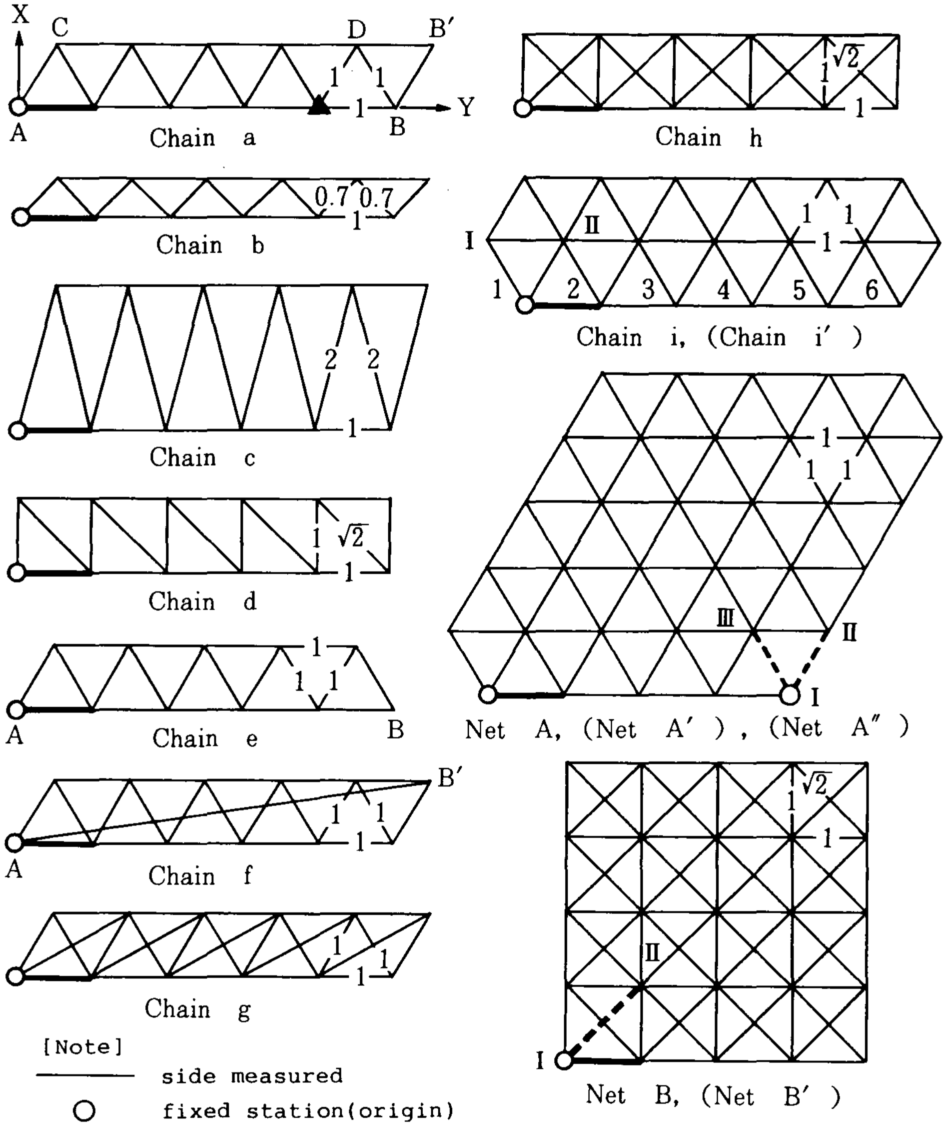
Among many subjects to be considered, characteristics of errors or accuracies of trilateration networks are exclusively dealt with in this paper. The method by condition equations is applied for adjusting observed data in spite of proposal of some new methods(9,10) because it was indicated in the previous paper(1) that this method was useful for inverstigating characteristics of propagation of observed errors. After foundamental properties of the errors in simple trilateration chains were inverstigated in that paper, error analyses of some more different types of survey chain and net have been carried out.

Various effects of redundant observations, constraint conditions and pattern or figure of trilateration nets are discussed in view of an accuracy in this paper. These data will be available for planning a control survey with small or middle scale.

2. SURVEYING NETS INVESTIGATED

Control survey networks by which the plane coordinates of many new stations are established, such as consruction survey for public and private works and location survey of highways, are treated. Main subjects considered in this paper are caracteristics of errors in trilateration networks, especially two types of network. The first ones are open networks which originate at a station of known position and terminate at a station of unknown position as shown in Fig.1. The second ones are closed networks which originate at a station of known position and close on another station of known position as shown in Fig.4.

Every chain and net extends lengthy to the Y direction rather than X direction as shown in Figs.1 to 4, and so the scale of a net is indicated by a number N of trilaterls connected along the Y direction instead of a total number of trilaterals constructing a whole net. Any station in the net is called by a number n which indicates a



[Note]

———	side measured
○	fixed station(origin)
━━━ / - - - -	fixed direction(Y direction)

[Note]

Chain e : side $\overline{A,B}$ is measured

Chain g, chain h and Net B : Diagonals are measured

Numerals written on sides show side length in L_0

Chain i' I:origin and $\overline{I,II}$:Y direction

Net A' I:origin and $\overline{I,II}$:Y direction

Net A" I:origin and $\overline{I,III}$:Y direction

Net B' I:origin and $\overline{I,II}$:Y direction

Fig.1 Open chains and nets

distance from the originating station to that station concerned as a general rue. But, in special case, a number of trilateral elements connected as far as that station is utilized.

3. ASSUMPTIONS AND METHOD OF COMPUTATION

As the object of this paper is to present caracteristics of errors in several typical trilateration networks, the simple types of figure and constraint condition, which are illustrated in Figs. 1 and 4, are selected. The following assumptions are, moreover, introduced in order to find out a general view of error propagation and evaluate an accuracy of a trilatelation project in preliminary stages of a work.

i) The most of sides in the trilateration nets are same length, and this length is adopted as a standard length of the sides constructing the nets. Therefore, That length is denoted by L_0 and is used for a unit of length. It follows, in the result, that the length of every side in the nets is unity with a few exceptions.

ii) The standard error σ_0 of an observation of side length is constant in spite of the length of a side with a few exceptions and denoted by $\varepsilon_0 L_0$ (ε_0 is a dimensionless constant).

iii) Each side length is measured independently.

According to the above assumptions, a propagated error σ in an estimated value is expressed by use of the unit σ_0 in case of lengths and coordinates, or $\sigma_0/L_0 = \varepsilon_0$ in case of angles and directions. Another measure of an accuracy, other than σ, is a cofactor Q. Cofactor Q is frequently used for explaining a pattern of propagation of observed errors in this paper. The variance σ^2 of an estimated value is computed according to the equation

$$\sigma^2 = Q\sigma_0^2 = Q\varepsilon_0^2 L_0^2 \qquad (1)$$

when an observed error σ_0 is known and the cofactor Q is computed.

The cofactors for coordinates of stations are denoted by $Q_{\hat{x}\hat{x}}$ and $Q_{\hat{y}\hat{y}}$, and the following quantity Q_{PP} are used for expressing the cofactor for planimetric position of a station.

$$Q_{PP} = Q_{\hat{x}\hat{x}} + Q_{\hat{y}\hat{y}} \qquad (2)$$

The method of least squares were applied to the trilateration networks illustrated in Figs. 1 and 4 under the above assumptions. Computational procedure was described in the previous paper. A process of propagation of the observed length errors to the directions of the sides and the coordinates of the stations was found easily by applying the method of condition equations compared with the method by observation equations. Another merit is that this method is suitable for use of an usual personal computer.

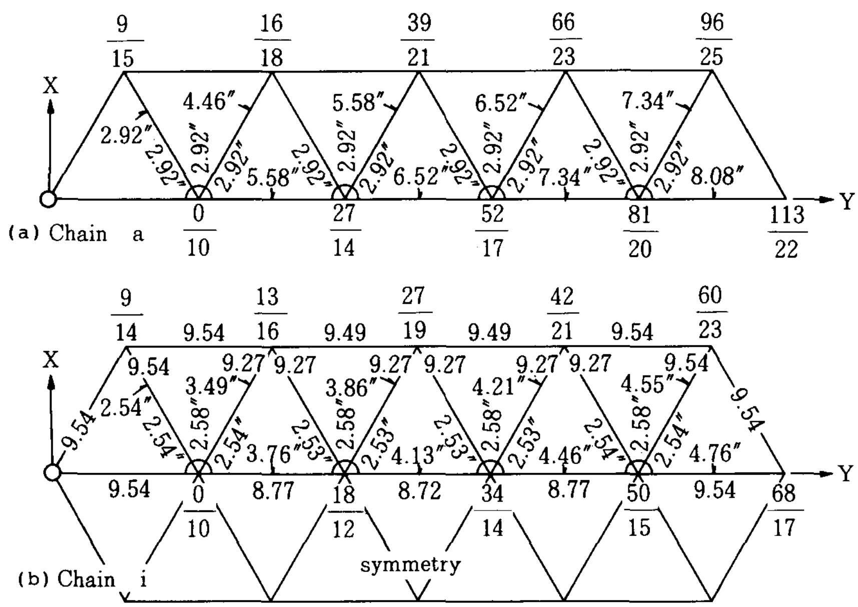
4. ERRORS IN TRILATERTION NETS WITHOUT EXTERNAL CONSTRAINT

A local plane rectangular coordinate system is introduced for adjustment computations as it is illusrated in Figs.1 to 3. The origin of the coordinate system is chosen at the station of known position and the Y axis is directed along a line passing through the origin.

4.1 Error Propagation in Two Types of Chains

The caracteristics of errors in trilatelation networks obtained from the previous paper are as follows. If trilaterals are connected in a figure of a single chain, $Q_{\hat{X}\hat{X}}$ for the successively connected stations from the origin increases according to a expression with a cubic function of n. To reduce $Q_{\hat{X}\hat{X}}$, it is remarkably effective to make up double row of single chains, that is to construct a single hexagonal chain. Though a hexagonal chain has only a small number of redundant observations, $Q_{\hat{X}\hat{X}}$ for the stations in the chain is well reduced due to strong constraint. On the contrary, in a single chain, $Q_{\hat{X}\hat{X}}$ is not so much reduced by redandant observations.

The errors in the single row and double row of chain are compaired in Fig.2 in terms of standard error σ instead of cofactor Q, in which the values of errors are computed under the assumption $\sigma_0=10\text{x}10^{-6}L_0$ or $\varepsilon_0=10\text{x}10^{-6}$. It is evident from Fig.2 that the standard errors of angles are not so much different in the both chains, but the errors of directions of sides of the single row chain successively increase as the sides are apart from the origin. It results in remarkably large errors of the coordinate X of stations in that chain. In the double row chain, on the other hand, these errors are fairly small.



[Note]

9.54,9.49,etc : Errors of side lengths.(unit.: $10^{-6}L_0$)

2.92" : Error of angle. 4.46" : Error of direction

$\frac{9}{14}$, $\frac{96}{25}$,ect : Numerators: errors of coordinates $\hat{X}$. (unit.: $10^{-6}L_0$)

: Denominators: errors of coordinates $\hat{Y}$. (unit.: $10^{-6}L_0$)

Fig.2 Errors of side lengths, angles, directions and coordinates of stations in the chain a and i', under the assumption $\sigma_0=10\times10^{-6}L_0$

4.2 Influences of Figures of Trilateral Elements

Chains a,b,c and d in Fig.1 are different in figure of trilateral elements. The errors in these chains are represented here in terms of cofactors instead of standard errors. Example of $Q_{\hat{X}\hat{X}}$, $Q_{\hat{Y}\hat{Y}}$ and Q_{PP} for the station with a distance $5L_0$ from the origin along the Y axis is summerized in Table 1. The station concerned is shown as station B in the chain a in Fig.1 for example. Meaning of the cofactors is explained in Eqs. (1) and (2). Table 1 tells us that $Q_{\hat{X}\hat{X}}$ decreases markedly as a height of trilateral elements incleases, and $Q_{\hat{Y}\hat{Y}}$ has, nevertheless, the same value through all chains. These characteristics do not appear in triangulation chains. These are the unique characteristics in the trilateral chains, but we must call attention to the assumption that the observed errors of lengths of every side are the same regardless of the differences of side lengths.

Table 1 Cofactors $Q_{\hat{x}\hat{x}}$, $Q_{\hat{y}\hat{y}}$ and Q_{pp} of stations with distance $5L_0$ in single row chains

	Chain a	Chain b	Chain c	Chain d
$Q_{\hat{x}\hat{x}}$	127	311	57.3	120
$Q_{\hat{y}\hat{y}}$	5	5	5	5
Q_{pp}	132	316	62.3	125

4.3 Influences of Internal Constraints

To investigate the effects for accuracy improvement by providing internal constraint in trilateration networks, redundant observations are applied to the several sides which are not necessary to construct fundamental trilateration nets.

Relations between the values of the cofactors for coordinates of stations and the conditions of redundant observations are described in Table 2. The six chains in the cases ① to ⑥ in Table 2 are single row chains with internal constraint. In these cases, the lengths to which redundant observations are applied are different in each other as shown in Table 2. It is, therefore, assumed that the standard errors of the observed sides are proportional to the square root of that side length or zero as special examples. The details of these assumptions are also given in the Table. For comparison with the previous data, the position of the stations listed in Table 2 is the same as in Table 1.

Table 2 Cofactors $Q_{\hat{x}\hat{x}}$, $Q_{\hat{y}\hat{y}}$ and Q_{pp} of stations with distance $5L_0$

Case	Chain	Redundant observation	Observed errors	No.of redundancy	$Q_{\hat{x}\hat{x}}$	$Q_{\hat{y}\hat{y}}$	Q_{pp}
	a	———	———	0	126.7	5.0	131.7
①	e	$\overline{A,B}$	$\sqrt{\overline{A,B}}$	1	113.3	2.5	115.8
②	a	$\overline{A,B}$, $\overline{C\ D}$	$\sqrt{\overline{A,B}}$, $\sqrt{\overline{C,D}}$	2	96.7	2.5	99.2
③	f	$\overline{A,B'}$	$\sqrt{\overline{A,B'}}$	1	125.5	4.3	129.8
④	e	$\overline{A,B}$	0	1	100.0	0	100.0
⑤	a	$\overline{A,B}$, $\overline{C,D}$	0	2	66.7	0	66.7
⑥	a	$\overline{A,B}$, $\overline{A,D}$, $\overline{C,B}$	$\sqrt{\overline{A,B}}$, $\sqrt{\overline{A,D}}$, $\sqrt{\overline{C,B}}$	3	111.4	2.3	113.7
⑦	g	diagonals	1	5	105.7	4.5	110.2
⑧	i'	margenal sides	1	4	46.7	2.8	49.5

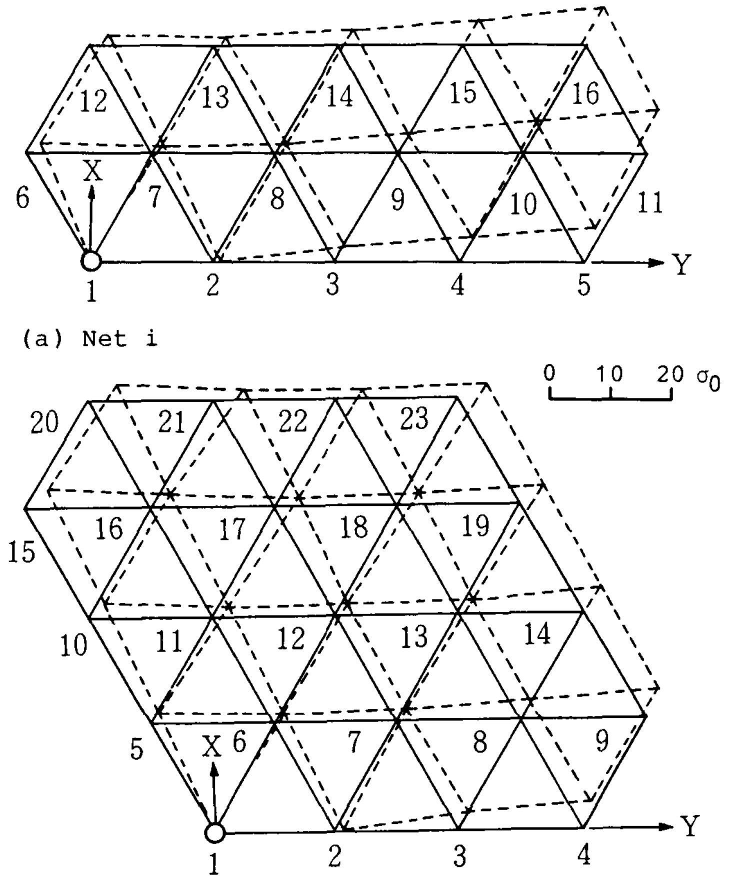
It will be evident from Teble 2 that
i) case ① is scarecely effective,
ii) excellent improvement of accuracy can not be expected from observations of diagonal-type long distances as shown in the cases ③ and ⑥,
iii) observations of two long parallel distance are effective a little as shown in the case ②, and
iv) the case ④ and ⑤ show that precize measurements of reduundant long distance are fairly effective.

Another case for providing internal constraints is to compose a rectangular chain by measuring every diagonal as shown in chains g and h. The one of the results is described as the case ⑦ in Table 2. This chain has five redundant observations, but the accuracy is not so well improved because the constraint condition is effective only for connecting the neighbouring two trilaterals.

In Fig.2(b), On the contrary, an example in which observations of short redundant sides provide an effective constraint for reduceing the errors of coordinates of stations in a chain is shown. That chain is a double row chain or hexagonal chain. The case ⑧ in Table 2 is the same chain as shown in Fig.2(b). $Q_{\hat{x}\hat{x}}$ in the case ⑧ is less than a half of that in case ⑦. The above facts tell us that it is improtant to provide closing conditions for many successive elements instead of each neighbouring element.

4.4 Positional Errors of Stations

In order to explane propagation of observed errors to the coordinates of stations, standard errors $\sigma_{\hat{x}}$ and $\sigma_{\hat{y}}$ of stations in two types of net are illustrated in Fig.3. It is found that the pattern of coordinate errors of stations is simillar in the two nets, the double row chain and the quadruple one. For more detail comparison, in Table 3, cofactors for the two nets are shown by arranging the corresponding points in a same row. Large differences between the cofactors for the coordinates of stations of the two types of net are not found. Therefore, it may be recognized that composing the double row of simple trilateration chains is considerably effective for decreasing the postional error of stations, but composing the triple or quadruple row are not so effective.



(a) Net i

(b) Net A'

Fig.3 Positional errors of stations in two types of open nets

4.5 Index of Strength of Figures

The purpose of plane trilatertion networks is to determine coordinates of each station in the nets rather than to know angles between sides or lengths of sides. Angles and side lengths are computed by comparably simple expressions from observed lengths and moreover they are little correlated with the observed lengths in case of internal constraint. Then, it results that the errors of them reveals themselves as fairly simple patterns shown in Fig.2. On the other hand, as the coordinates of stations are computed by successive summation functions of angles and lengths, the errors of the coordinates inclease progressively as the stations are apart from the origin of the net.

On the base of the above considerations, cofactor Q_{pp} is used for

Table 3 Cofactors of stations in the nets i and A'

Net i				Net A'			
St.	$Q_{\hat{x}\hat{x}}$	$Q_{\hat{y}\hat{y}}$	Q_{PP}	St.	$Q_{\hat{x}\hat{x}}$	$Q_{\hat{y}\hat{y}}$	Q_{PP}
2	0	0.9	0.9	2	0	0.9	0.9
3	5.5	1.8	7.3	3	5.2	1.6	6.8
4	16.1	2.6	18.7	4	14.1	2.4	16.5
5	34.0	3.5	37.5	—	—	—	—
6	2.1	3.0	5.1	5	2.1	3.0	5.0
7	0.6	2.2	2.7	6	0.5	2.1	2.7
8	2.1	2.8	4.9	7	2.0	2.8	4.8
9	9.5	3.0	12.6	8	8.6	3.0	11.6
10	23.6	3.4	27.0	9	20.0	3.6	23.6
11	45.7	4.0	49.7	—	—	—	—
12	1.6	7.0	8.6	11	1.4	6.9	8.3
13	1.6	6.9	8.5	12	1.4	6.8	8.2
14	5.4	7.9	13.3	13	5.0	7.4	12.3
15	16.0	8.7	24.8	14	13.4	7.9	21.3
16	34.0	9.6	43.6	—	—	—	—
				20	6.3	24.9	31.2
				21	2.3	24.3	26.6
				22	2.3	24.3	26.6
				23	6.0	24.7	30.8

evaluating a relative accuracy between the nets in this paper. That is, we assume that strength of figure of a net can be represented by the value of Q_{PP} for a station. The station with a distance $4L_0$ from the origin is selected for comparing the values of Q_{PP} in the nets. The positions of the stations are illustrated by a solid triangles in Fig.1.

The values of Q_{PP} for those stations in typical types of single chain and net are shown in Table 4. Average of cofactors for estimated lengths in a net are also described in this Table by denoting as $\overline{Q}_{\hat{l}\hat{l}}$. $\overline{Q}_{\hat{l}\hat{l}}$ equals to (m-r)/m in which m is a total number of observations and r means a number of redundant observations.

It is known from Table 4 that

i) as efficiency of redundand observations for reduceing the positional error of stations varies from case to case, r merely represents a degree of redundancy and is nearly independent of strength of internal constraint or strength of figure in a net, and

ii) Table 4 may be considered as a summary of the data from Tables 1 to 3, then it is helpful for getting a knowledge on characteristics of accuracy of the nets at a glance.

Table 4 Relative strength in open chains and nets

Net	r	$Q_{\hat{l}\hat{l}}$	Q_{pp}
a	0	1	69.3
b	0	1	157.2
c	0	1	36.3
d	0	1	68.0
e	1	0.950	62.9
f	1	0.955	68.3
g	5	0.808	56.5
h	5	0.808	43.5
i	5	0.875	37.5
i'	5	0.875	27.8
A	16	0.802	31.3
A'	16	0.802	29.8
A"	16	0.802	19.8
B	25	0.653	24.9
B'	25	0.653	12.3

Table 5 $\bar{Q}$ and Q_{max} in open chains and nets

Net	N	$\bar{Q}$	Q_{max}
a	3	4.8	9.3
	5	11.5	30.3
	7	23.4	69.3
	9	41.8	132
	10	53.9	175
	11	68.0	223
b	3	6.6	16.4
	5	20.8	62.9
	7	48.2	157
	9	93.0	316
	10	123	425
c	3	6.9	10.9
	5	10.8	19.3
	7	16.9	36.3
	9	25.3	62.3
	10	30.9	86.4
d	3	5.5	10.0
	5	11.3	31.0
	7	21.5	68.0
	9	37.0	125
	10	45.5	130
	11	58.5	206
e	3	4.3	8.0
	5	10.6	26.8
	7	21.0	55.2
	9	38.2	116
	13	93.3	302
	17	185	620
f	2	2.6	4.5
	6	16.4	47.8
	8	31.2	97.5
	10	53.0	173
	12	83.1	279
	16	173	605
g	2	2.4	4.1
	4	5.8	14.3
	6	13.0	38.3
	8	25.4	80.7
	10	44.1	147

Net	N	$\bar{Q}$	Q_{max}
h	2	2.2	2.9
	4	3.9	7.1
	6	8.3	19.9
	8	16.3	44.6
	10	28.7	85.2
i	3	5.2	8.6
	5	7.1	13.3
	7	11.2	27.3
	9	17.7	49.7
	11	26.8	81.4
i'	3	3.4	5.0
	5	6.1	13.4
	7	10.5	27.8
	9	17.1	49.3
	11	26.1	79.8
A	3	5.2	8.6
	5	11.1	25.1
	7	21.2	56.2
	9	35.7	102
A'	3	5.2	8.6
	5	9.7	17.7
	7	15.8	31.2
	9	23.5	47.7
A"	3	3.4	5.0
	5	6.2	10.7
	7	10.7	20.4
	9	16.3	32.7
B	2	2.2	2.9
	4	5.4	10.5
	6	10.6	24.9
	8	17.8	46.1
B'	2	1.5	1.8
	4	2.7	4.6
	6	5.4	12.4
	8	9.6	25.5

4.6 Estimation of Cofactors for Coordinates of Stations

Considerations on cofactors for coordinate of stations in a survey network are the most important subject to make a plan of a control survey and, as described in the previons sections, these cofactors vary strikingly with the figure of the net and internal constraints. Taking into account this fact, mean values and the maximum values of Q_{pp} are summerized in Table 5 by use of the notations $\bar{Q}$ and Q_{max}, respectively.

$Q_{\hat{x}\hat{x}}$ of any stations on the Y axis in the chain a increases in accordance with the third order polynomials of a distanse, or the number of connecting trilaterals, from the origin to that stasion. It is because the angles of any triraterals are independent of every observed length in the chain, which will be supposed from the characteristics of positional errors of any stations in a open traverse. Every single row chain hase this characteristic. When effectiveness of internal constraint is weak even if redundant observations are applied to a single row chain, the tendency of increase of errors is approximately same as above.

In the chain i and i', the angles and directions are not so strongly correlated with the lengths of the sides which are far from them, though they are closely correlated with the lengths of the sides near themselves. This is the reason why $Q_{\hat{x}\hat{x}}$, hence Q_{PP}, increases nearly in accordance with the third order polynomials of a distance from the origin, or station number n, as shown in Fig.2, Tables 3 and 5.

In the planimetrically spreaded nets such as the nets A and B in Fig.1, the values $Q_{\hat{y}\hat{y}}$ are nearly same as those of $Q_{\hat{x}\hat{x}}$, and the both are fairly small. It is an outstanding property of widely extended nets. We can find, in Table 5, that Q_{max} also increases approximately in accordance with the third order polynomials of N, and $\overline{Q}$ in accordance with the second order polynomials. On the base of these tendencies, the cofactors Q_{max} and $\overline{Q}$ of larger nets than the computed ones will be able to be estimated by use of Table 5.

5. ERRORS IN TRILATELATION NETS WITH EXTERNAL CONSTRAINT

Trilatertion networks illustrated in Fig.4 are selected as typical examples of network with external constraint, all of which originate at a station of known position and close on another station of known position. These nets have the same figures as shown in Fig.1. Notation of the nets is refered to Fig.4, that is, subscript 0 is attached to the notation in Fig.1 for distinguishing.

The one of given stations is chosen as the origin of coordinate system and the line connecting the two given stations is chosen as the Y axis in principle, as illustrated in Fig.4.

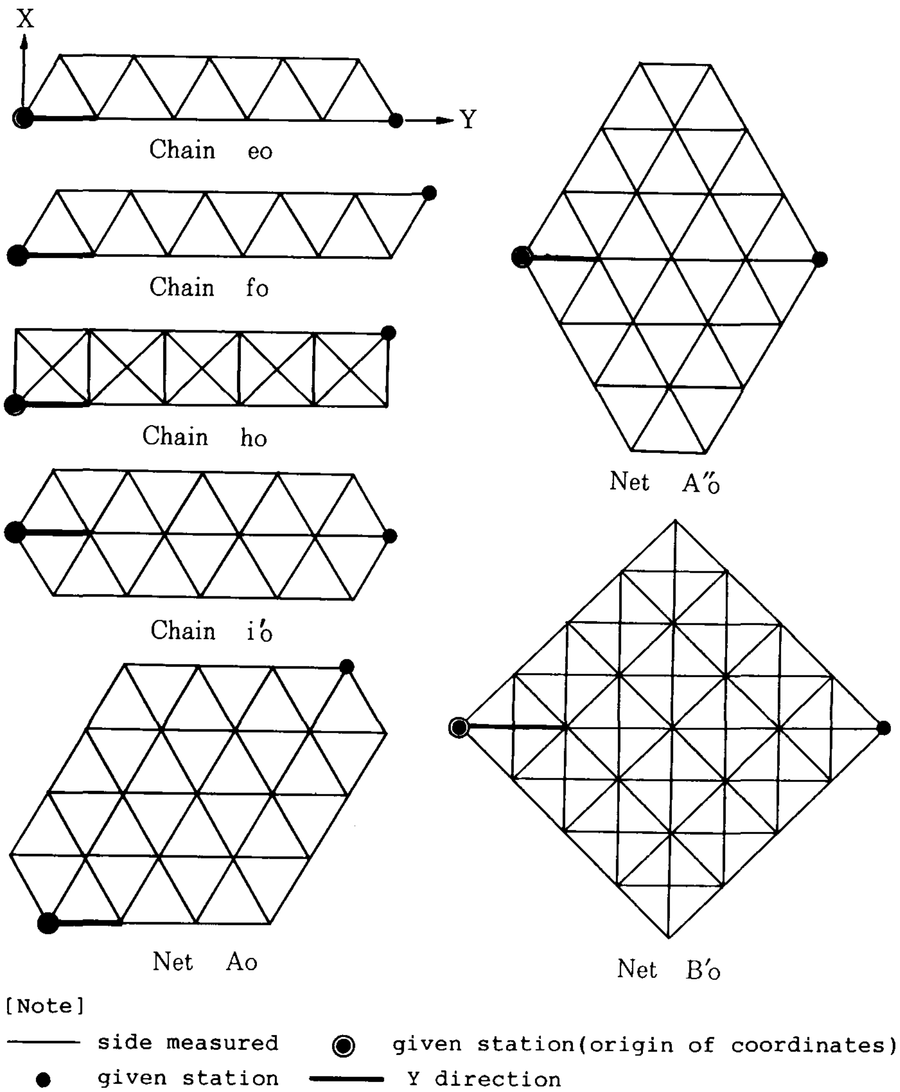


Fig.4 Closed chains and nets

5.1 Error Propagation in Several Closed Nets

Errors and cofactors for a single row of trilateral chain e_0 composed of 17 elements shown in Fig.5 are computed as an example. Q_{PP} for the stations in the chain are given in Table 6, in which the notation of stations is also shown in Fig.5. Q_{PP} is fairly smaller than the one in the open chain e shown in Table 2. Increasing rate of Q_{PP} in accordance with station number n or distance are very small and Q_{PP} increases nearly proportionally to square root of n. From the fact, It may be readily imagined that even if a long chain with single row trilaterals is constructed between two given stations, the maximum error of positions of stations in the chain is not so large.

The difference in effectiveness of two types of constraint, internal and external, may be recognized distinctly by inspecting the following simple example. We assume here that every side of trilateral elements of chain is L_0=200m and observed error of the side length is $\sigma_0=5mm=25x10^{-6}L_0$. Then We can obtain the following results. When two given points with a distance 1,800m are connected by the chain e_0 which is composed of 17 tritalerals or 17 new stations, the maximum positional error of stations will be $\sqrt{33.1}\,\sigma_0=29mm=144x10^{-6}L_0$ according to Table 6. In the cases of the chains a and e, on the otherhand Table 5 shows that the maximum positional error attains to $\sqrt{69.3}\,\sigma_0=42mm=208x10^{-6}L_0$ and $\sqrt{55.2}\,\sigma_0=37mm=186x10^{-6}L_0$, respectively, even if these chains are composed of 7 trilateral elements only.

Another example is the case of a net A''_0, expanded planimetrically. Positional errors σ_x and σ_y, insted of Q, are shown in Fig.6 for illustrating propagation of observed errors. It is evident that these positional errors increase with distance from the fixed stations but are remarkably smaller than those in Net A' illustrated in Fig.3(b). External constraint is also effective for such a net.

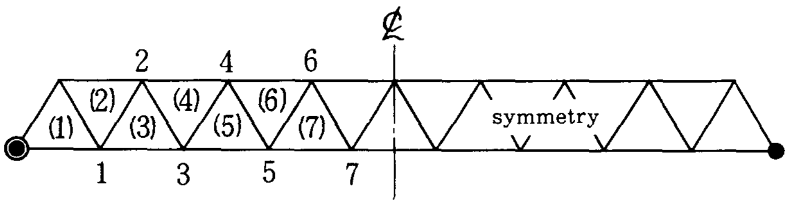


Fig.5 Station numbers in the closed chain e_0 with N=17

Table 6 Q_{PP} of stations in the chain e_0

Station	1	2	3	4	5	6	7
Q_{PP}	7.3	13.8	18.0	23.1	27.6	30.3	33.1

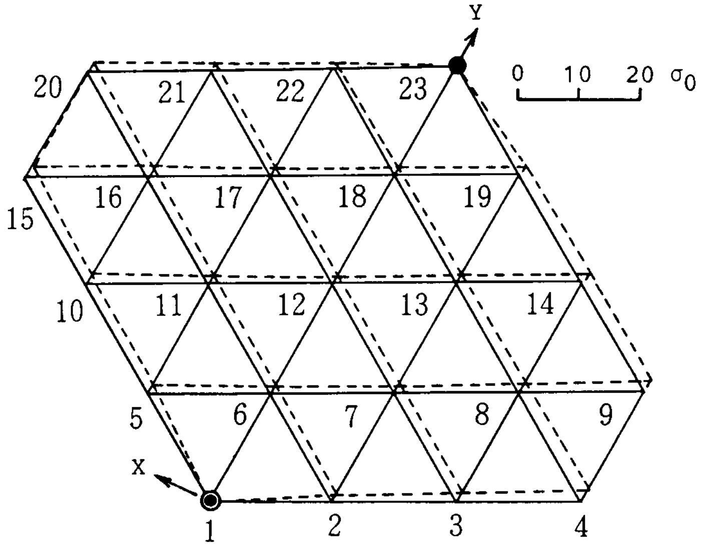


Fig.6 Positional errors of stations in the closed Net A"$_0$

5.2 Posisional Errors and Influences of External Constraints

Relations between cofactors and number N of trilateral elements composing the chains are summerized in Table 7. In this Table, $\overline{Q}$ and Q_{max} mean an average and the maximum of Q_{pp}, respectively. As single row chains e_0 and f_0 are very "flexible" as described in Table 2, the maximum errors in those types of chain are fairly larger than the ones in other types even if the both end stations of the chains are closed on given positions.

Table 7 Q and Q_{max} in closed chains

Ch.	N	$\overline{Q}$	Q_{max}	Ch.	N	$\overline{Q}$	Q_{max}
e_0	3	2.4	2.5	h_0	2	1.7	1.7
	5	3.2	4.0		4	2.0	2.3
	9	6.4	8.8		6	2.7	2.9
	13	11.9	17.9		8	3.9	4.9
	17	20.6	33.1		10	5.6	7.7
f_0	6	4.7	6.0	i'_0	3	2.1	2.3
	8	7.0	9.3		5	2.3	2.8
	10	10.0	14.7		7	2.9	3.5
	12	14.1	21.4		9	3.7	4.9
	16	26.1	42.2		11	4.7	6.4

Table 8 shows the comparison of the values of $\overline{Q}$ as well as Q_{max} in the nets with two types of constraints, internal and external constraints. It is evident that splendid efficiency for reducing the maximum positional errors in a chain is obtained by external constraint, especially in the case of single trilateral chains. By comparison between nets A_0 and A''_0 which are extented planimetrically, it is found that external constraint is more effective to A_0 because it is constrained through a longer distance.

Table 8 Comparison of $\overline{Q}$ and Q_{max} between open and closed nets

Open nets				Closed nets		
N	Net	$\overline{Q}$	Q_{max}	Net	$\overline{Q}$	Q_{max}
9	e	38.2	116	e_0	6.4	8.8
10	f	53.0	173	f_0	10.0	14.7
10	h	28.7	85.2	h_0	5.6	7.7
9	i'	17.1	49.5	i'_0	3.7	4.9
7	A	21.2	56.2	A_0	3.1	4.2
7	A"	10.7	20.4	A''_0	3.1	5.1
8	B'	9.6	25.5	B'_0	2.8	5.1

6. CONCLUDING REMARKS

Characteristics of errors in trilateration nets are systematically explained by use of simple and foundamental figures of net such as single row, double row and quadruple row, as well as several constraint conditions. The reason why these subjects are investigated in this paper is that they are useful for making a new plan for control survey.

The summerize are as follows:

i) Errors of a single row of chain increase seriously in accordance with its length. To reduce them it is effective to compose a double row of chain.

ii) Errors of a single row of chain which is composed of elements with small hight are fairly greater than the one composed of those with large hight. This is a different characteristic from the case of triangution.

iii) Internal contraints enforced to a single row of chain are not so

effective to reduce the error in the chain.
iv) External constraints are useful for reducing the error in any trilateration chains and nets.

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