

## *Mean Time Between Failure of Ring Arbiter with Requests Differing in Incidences*

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### SYNOPSIS

In asynchronous arbiters, failures may happen, caused by metastable operations. The purpose of this study is to derive a formula to estimate such failures in a ring arbiter as mean time between failures (MTBF), under the condition that incidences of requests issued in all devices are different from each other. The operation of the arbiter is formularized by a markov chain. This chain is used to decide the probability at which each of possible failures contributes to MTBF. The sum of such probabilities gives the MTBF which can be represented as a sum of a finite number of terms. As an example, MTBF of a ring arbiter composed of 3 cells is shown.

### 1. INTRODUCTION

In computer systems, a resource is shared by two or more devices and conflicts may occur for its exclusive use. In order to resolve such conflicts, arbiters can be often used. In particular, an arbiter which allows devices to issue requests asynchronously is called an asynchronous arbiter. All asynchronous arbiters have a function to compare times of two or more signals occurring asynchronously. In most asynchronous arbiters, flip-flops (hereafter simply referred to FF) have been used to realize such this function<sup>[1]-[4]</sup>. If such times are critically near, metastable operations (hereafter referred to as MSO) occur in FFs, and in the worst case, failures may happen in any asynchronous arbiter<sup>[5]</sup>. Thus, it is a very important problem to develop useful ways to estimate such failures.

We already proposed one of ways to estimate failures as the mean time between failures (hereafter simply referred to as MTBF)<sup>[6]-[8]</sup>. The MTBF can be estimated by the use of a formulas, which are derived from the markov chains represented the behaviors of asynchronous arbiters. In these formulas, however, it is assumed that incidences of requests in all devices are identical.

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In this study, we show a formula to estimate MTBF of a ring arbiter under the condition that incidences of requests in all devices are different from each other. First, the operation of this is formularized as a markov chain. Second, by the use of the markov chain the formula for estimating MTBF is derived. Finally, examples of estimating MTBF are shown.

## 2. MARKOV CHAIN OF RING ARBITER

### 2.1 Preconditions

A ring arbiter (hereafter referred to as RA) consists of a number of identical cells  $C_i$ s ( $1 \leq i \leq n, n \geq 2$ :  $n$  is the number of devices) connected in ring structure, as shown in Fig. 1.  $r_i$  and  $A_i$  are the request signal and the acknowledgment signal in  $C_i$ , respectively. The value of  $r_i$  is logically high (hereafter referred to as 1) iff the device  $i$  issues the request to  $C_i$ , otherwise it is logically low (hereafter referred to as 0). The value of  $A_i$  is 1 iff  $C_i$  issues the acknowledgment to the device  $i$ , otherwise  $A_i = 0$ .

In RA, there is exactly one privilege, which circulates along the direction of arrows on it.(see Fig.1) When the privilege arrives at  $C_i$ , if  $r_i = 0$ , it is immediately transferred to  $C_{i+1}$ . Here and in the following, the operation of the subscript  $i$  should be taken modulo  $n$ . On the other hand, when it arrives at  $C_i$ , if  $r_i = 1$ , the corresponding request is acknowledged ( $A_i = 1$ ), and it is transferred to  $C_{i+1}$  after the device  $i$  completes the use of the resource. If the arrival time of the privilege to  $C_i$  gets close to the time when the request issues, a metastable operation (hereafter referred to as MSO) occurs in  $C_i$  and in the worst case a failure may happen in RA. In this paper, MTBF (Mean Time Between Failures) on such a failure is estimated.

In order to estimate MTBF, structures (including properties with respect to MSO) of cells and devices must be known. Hence, in this paper, following assumptions are set up.

- [Assumption 1] The time required for each device to use the resource on once arrival of the privilege is constant.

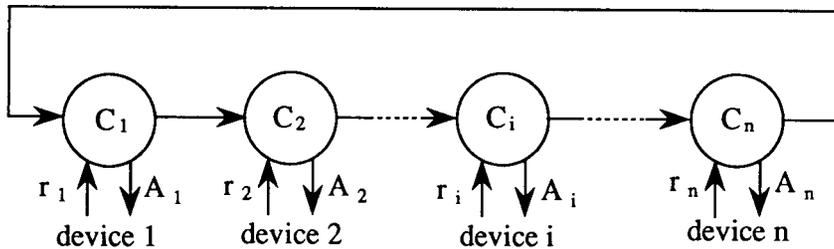


Fig. 1 Block diagram of ring arbiter.

- [Assumption 2] Any failure never happens in any device.
- [Assumption 3] Structures of cells are identical.
- [Assumption 4] Propagation delay times on wires interconnecting cells and/or devices are zero.
- [Assumption 5] The duration time of MSO is zero.
- [Assumption 6] Each cell has a flip-flop (hereafter referred to as FF) to resolve the conflict of the two times, the privilege arrival and the request issuing. MSO can be occurred only in this FF.
- [Assumption 7] The following probabilities are known.
- (1) The probability that the request is acknowledged without any failure, when the privilege arrives at  $C_i$  and MSO occurs. ( $W_A$ )
  - (2) The probability that the request is not acknowledged without any failure, when the privilege arrives at  $C_i$  and MSO occurs. ( $1 - W_E - W_A$ )
  - (3) The probability that any failure occurs, when the privilege arrives at  $C_i$  and MSO occurs. ( $W_E$ )

Under above assumptions, the operation of a cell is classified into following four operation modes (hereafter simply referred to as mode).

- [Mode 1] MSO never occurs and the request is never acknowledged.
- [Mode 2] Regardless of MSO occurring or not, the request is acknowledged without failures.
- [Mode 3] MSO occurs and the request is never acknowledged without failures.
- [Mode 4] MSO occurs and any failure occurs.

An example of failures in the mode 4 is a misoperation in which the request is acknowledged and the privilege is transferred to the next cell at the same time.

## 2.2 State Transitions of Ring Arbitrator

When the privilege is transferred to cell  $C_i$ , the probability of the mode occurring depends on those of all cells (including  $C_i$ ) when the privilege visits them last time, because the probability that each device issues the request depends on the duration time took for the privilege to circulate the ring last time. Consequently, in this paper the state of RA is defined as follows.

Let  $m_i$  denote the mode of  $C_i$  at present time. If  $m_j$  ( $1 \leq j \leq n, j \neq i$ ) is the mode of  $C_j$  at last privilege visitation of  $C_j$ . The probability of next mode of  $C_{i+1}$  occurring is determined

by the sequence of modes  $m_{i+1}, \dots, m_n, m_1, \dots, m_i$ , where  $1 \leq m_j \leq 3$ ,  $1 \leq m_i \leq 4$ ;  $1 \leq j \leq n$ ,  $j \neq i$ . Thus, we define the state of RA by this sequence. The occupation time of a state  $m_{i+1} \dots m_n m_1 \dots m_i$  is the same as the time when it takes for the privilege pass through  $C_i$  on mode  $m_i$ . Furthermore, the transition probability that the state changes from  $m_{i+1} \dots m_n m_1 \dots m_i$  to  $m_{i+2} \dots m_n m_1 \dots m_i m'_{i+1}$  is written by  $B_{i+1}(m'_{i+1} | m_{i+1} \dots m_n m_1 \dots m_i)$ ,  $m'_{i+1}$  means the operation mode, when the privilege visits  $C_{i+1}$  next time. If the configuration of RA is given, values of  $B_{i+1}$ s and these occupation times can be calculated under the assumption 1 ~ 7. All over the operation of RA can be represented by a markov chain with  $n \cdot 4 \cdot 3^{n-1}$  states.

It is easily clarified that there exists a stationary state in this markov chain<sup>[6]</sup>. Let  $S_i(m_{i+1} \dots m_n m_1 \dots m_i)$  and  $S_{i+1}(m_{i+2} \dots m_n m_1 \dots m_i m'_{i+1})$  denote incidence probabilities of the state  $m_{i+1} \dots m_n m_1 \dots m_i$  and  $m_{i+2} \dots m_n m_1 \dots m_i m'_{i+1}$ , in the stationary state, respectively. Then, following equations hold from the definition of the stationary state,

$$\begin{aligned} & \sum_{m_{i-1}=1}^3 S_i(m_{i+1} \dots m_n m_1 \dots m_i) \cdot B_{i+1}(m'_{i+1} | m_{i+1} \dots m_n m_1 \dots m_i) \\ & = S_{i+1}(m_{i+2} \dots m_n m_1 \dots m_i m'_{i+1}), \end{aligned} \quad (1)$$

$$\sum_{i=1}^n \sum_{K_i} S_i(m_{i+1} \dots m_n m_1 \dots m_i) = 1, \quad (2)$$

where  $\sum_{K_i}$  shows the total sum about  $S_i$  under the condition that the privilege stay  $C_i$ . Thus, from  $n \cdot (3^n - 1)$  equations that take the same form of the equation (1), and the equation (2), all state incidence probabilities can be obtained.

Let  $M_i(m'_i)$  denote the probability that the cell  $C_i$  falls into mode  $m'_i$  at arrival of the privilege  $M_i(m'_i)$  is given from the definition by the following equation.

$$M_i(m'_i) = \sum_{K_{i-1}} S_{i-1}(m_i \dots m_n m_1 \dots m_{i-1}) \cdot B_i(m'_i | m_i \dots m_n m_1 \dots m_{i-1}). \quad (3)$$

Besides, under the condition that the privilege stays in  $C_{i-1}$  in the stationary state, the probability  $P$  that it passes through  $C_i$  without any failure and  $C_{i+1}$  falls into mode  $m'_{i+1}$  is given the following equation,

$$\begin{aligned} P & = \sum_{m'_i=1}^3 \sum_{K_{i-1}} S_{i-1}(m_i \dots m_n m_1 \dots m_{i-1}) \\ & \cdot B_i(m'_i | m_i \dots m_n m_1 \dots m_{i-1}) \cdot B_{i+1}(m'_{i+1} | m_{i+1} \dots m_n m_1 \dots m'_i) \\ & = \left\{ \sum_{m_{i-1}=1}^3 M_{i-1}(m_{i-1}) \right\} \cdot \left\{ \sum_{K_i} S_i(m_{i+1} \dots m_n m_1 \dots m_i) \right. \\ & \cdot \left. B_{i+1}(m'_{i+1} | m_{i+1} \dots m_n m_1 \dots m'_i) \right\}. \end{aligned} \quad (4)$$

### 3. FORMULA OF ESTIMATING MTBF

Let us select the starting time of estimating MTBF to the time ( $t_0$ ) when the privilege is transferred to cell  $C_i$  under the condition that RA stays at the stationary state. It is assumed that the privilege has passed through  $x$  cells without failures after the time  $t_0$ , modes of which are  $\bar{m}_1, \bar{m}_2, \dots, \bar{m}_j, \dots, \bar{m}_x, 4$  ( $1 \leq \bar{m}_j \leq 3, 1 \leq j \leq x$ ) respectively, and that the failure just happens in the next cell. Note that  $\bar{m}_j$  means the mode of the cell to which the privilege reaches  $j$ th cell. The probability  $E_i(\bar{m}_1 \bar{m}_2 \dots \bar{m}_x 4)$ , that the mode chain  $\bar{m}_1 \bar{m}_2 \dots \bar{m}_j \dots \bar{m}_x 4$  happens, is given by,

$$\begin{aligned}
 E_i(\bar{m}_1 \bar{m}_2 \dots \bar{m}_x 4) &= \sum_{K_{i-1}} S_{i-1}(\bar{m}_{1-n} \bar{m}_{2-n} \dots \bar{m}_0) \\
 &\quad \prod_{k=0}^x B_{g(k,i)}(\bar{m}_{k+1} | \bar{m}_{1-n+k} \bar{m}_{2-n+k} \dots \bar{m}_k), \\
 &: \quad g(k,i) = \begin{cases} k - [k/n] \cdot n + i & (k - [k/n] \cdot n + i \leq n) \\ k - [k/n] \cdot n + i - n & (k - [k/n] \cdot n + i > n) \end{cases} \\
 &: \quad K_{i-1} = \{m_i \dots m_n m_1 \dots m_{i-1} | 1 \leq m_j \leq 3, 1 \leq j \leq n\}
 \end{aligned} \tag{5}$$

where  $g(k, i)$  is the number assigned to the cell which the privilege can reach through  $k - 1$  cells after  $t_0$ , and  $[\beta]$  is a gauss operation of  $\beta$ . And  $\bar{m}_{1-n} \bar{m}_{2-n} \dots \bar{m}_0$  is the state at  $t_0$ . On the other hand, from assumption 4 and 5, the time  $T_i(\bar{m}_1 \bar{m}_2 \dots \bar{m}_x 4)$  taken for the failure to happen in  $x + 1$ st cell, is given by,

$$T_i(\bar{m}_1 \bar{m}_2 \dots \bar{m}_x 4) = \sum_{j=1}^x T(\bar{m}_j), \tag{6}$$

where  $T(\bar{m}_j)$  is the duration that the mode of  $C_{g(j,i)}$  stays at  $\bar{m}_j$ . Therefore, from the equations (5) and (6), the contribution  $T_{MF_i}$  to MTBF under the condition mentioned above is given as follows;

$$\begin{aligned}
 T_{MF_i} &= \sum_{x=1}^{\infty} \sum_{K_x} T_i(\bar{m}_1 \bar{m}_2 \dots \bar{m}_x 4) \cdot E_i(\bar{m}_1 \bar{m}_2 \dots \bar{m}_x 4) \\
 &= \sum_{x=1}^{\infty} \sum_{K_x} \sum_{K_{i-1}} S_{i-1}(\bar{m}_{1-n} \bar{m}_{2-n} \dots \bar{m}_0) \\
 &\quad \cdot \left\{ \sum_{j=1}^x T(\bar{m}_j) \right\} \cdot \left\{ \prod_{k=0}^x B_{g(k,i)}(\bar{m}_{k+1} | \bar{m}_{1-n+k} \bar{m}_{2-n+k} \dots \bar{m}_k) \right\}
 \end{aligned}$$

$$\begin{aligned}
&= \sum_{j=1}^{\infty} \sum_{x=j}^{\infty} \sum_{K_x} T(\bar{m}_j) \sum_{K_{i-1}} S_{i-1}(\bar{m}_{1-n} \bar{m}_{2-n} \cdots \bar{m}_0) \\
&\cdot \left\{ \prod_{k=0}^x B_{g(k,i)}(\bar{m}_{k+1} \mid \bar{m}_{1-n+k} \bar{m}_{2-n+k} \cdots \bar{m}_k) \right\}. \tag{7}
\end{aligned}$$

And from the equation (4), the equation (7) can be transformed as follows;

$$\begin{aligned}
T_{MF_i} &= \sum_{j=1}^{\infty} \left\{ \prod_{h=1}^n \left( \sum_{m_h=1}^3 M_h(m_h) \right)^{f(j,h,i)} \right\} \sum_{x=1}^{\infty} \sum_{K_x} T(\bar{m}_1) \\
&\cdot \sum_{K_{g(j,i)}} S_{g(j,i)}(\bar{m}_{1-n} \bar{m}_{2-n} \cdots \bar{m}_0) \\
&\cdot \prod_{k=0}^x B_{g(k,g(j,i))}(\bar{m}_{k+1} \mid \bar{m}_{1-n+k} \bar{m}_{2-n+k} \cdots \bar{m}_k), \tag{8} \\
&: f(x, h, j) = [x/n] + 1 \quad : (\text{case } 1 \leq g(x, i) \leq i-1, 1 \leq h \leq g(x, i) \text{ or } i \leq h \leq n) \\
&\quad \quad \quad \quad \quad \quad : (\text{case } i \leq g(x, i) \leq n, i \leq h \leq g(x, i)) \\
&= [x/n] \quad \quad \quad : (\text{case } 1 \leq g(x, i) \leq i-1, g(x, i) < h \leq i-1) \\
&\quad \quad \quad \quad \quad \quad : (\text{case } i \leq g(x, i) \leq n, 1 \leq h \leq i-1 \text{ or } g(x, i) < h \leq n),
\end{aligned}$$

where  $f(x, h, j)$  is the times when the privilege arrives at  $C_h$  during  $T_i(\bar{m}_1 \bar{m}_2 \cdots \bar{m}_x)$ , and the term followed to  $\sum_{x=1}^{\infty}$  on the right side of the equation (8), is the mean time taken for the privilege to pass through  $C_{g(j,i)}$ . Therefore, from assumption 1, 3, 4 and the equation (3), the equation (8) can be transformed to the next equation.

$$\begin{aligned}
T_{MF_i} &= \sum_{j=1}^{\infty} \left\{ \prod_{h=1}^n \left( \sum_{m_h=1}^3 M_h(m_h) \right)^{f(j,h,i)} \right\} \\
&\cdot \{T_1(M_{g(j,i)}(1) + M_{g(j,i)}(3)) + (T_1 + T_2)M_{g(j,i)}(2)\}, \tag{9}
\end{aligned}$$

where  $T_1$  and  $T_2$  are the time required for the privilege to pass through one cell in the case of the the request acknowledged and not, respectively. Furthermore, rearranging the above equation in the respect to the values  $g(j, i) (1 \leq g(j, i) \leq n)$ ,

$$\begin{aligned}
T_{MF_i} &= \sum_{h=1}^n \left\{ \prod_{l=1}^h \left( \sum_{m_l=1}^3 M_l(m_l) \right) \right\} \left\{ \sum_{k=0}^{\infty} \left\{ \prod_{y=1}^n \left( \sum_{v=1}^3 M_{g(y,i)}(v) \right) \right\} \right\} \\
&\cdot \{T_1(M_{g(h,i)}(1) + M_{g(h,i)}(3)) + (T_1 + T_2)M_{g(h,i)}(2)\}, \tag{10}
\end{aligned}$$

can be derived. Finally,  $T_{MF_i}$  is given by,

$$T_{MF_i} = \sum_{h=1}^n \left\{ \prod_{l=1}^h \left( \sum_{m_l=1}^3 M_l(m_l) \right) \right\} \cdot \{T_1(M_{g(j,i)}(1) + M_{g(j,i)}(3)) + (T_1 + T_2)M_{g(j,i)}(2)\}$$

$$/ \{1 - \prod_{y=1}^n (\sum_{v=1}^3 M_{g(y,i)}(v))\}. \tag{11}$$

4. EXAMPLE

As an example, we estimated MTBF of a ring arbitrer which consists of three cells constructed as shown in Fig. 2. Let the propagation delay time of a NAND gate be 7(nsec), and let  $T_1$  and  $T_2$  be 14(nsec) and  $T_0 + 21(nsec)$ , respectively, where  $T_0$  denotes the time required for each device to use the resource each time. Furthermore, let  $W_E = 10^{-3}$ ,  $W_A = (1 - W_E)/2$ ,  $u_2 = \alpha_2 \cdot u_1$  and  $u_3 = \alpha_3 \cdot u_1$ , where  $u_i$  denote the request incidence of the device  $i$  ( $1 \leq i \leq 3$ ).

Fig. 3 and Fig. 4 show results of MTBF estimated, assuming that  $T_0 = 200(nsec)$ ,  $\alpha_2 = 1$  and  $T_0 = 300(nsec)$ ,  $\alpha_2 = 1$ , respectively. In both figures, relations between  $\alpha_3$  and  $F_M$  (normalized MTBF<sup>[9]</sup>), employing  $u_1$  as a parameter. According to these results, the value of  $F_M$  changes as follows. Let  $u_{max}$  be the maximum value among  $u_1 \sim u_3$  and let  $T_a$  be the mean of the time required for the privilege to circulate the ring. If the value of  $u_{max}$  is lower than  $1/T_a$ , the value of  $F_M$  is pretty small. On the contrary, it becomes higher than  $1/T_a$ , the value of  $F_M$  begins increasing remarkably.

5. CONCLUSION

In this study, we derived a formula to estimate MTBF of ring arbiters under the condition that request incidences of devices are different from each other. By the use of this formula, practical values of MTBFs can be calculated easily if constructions of ring arbiters and devices are given.

The procedure for deriving the formula can be also applied to that for other arbiters.

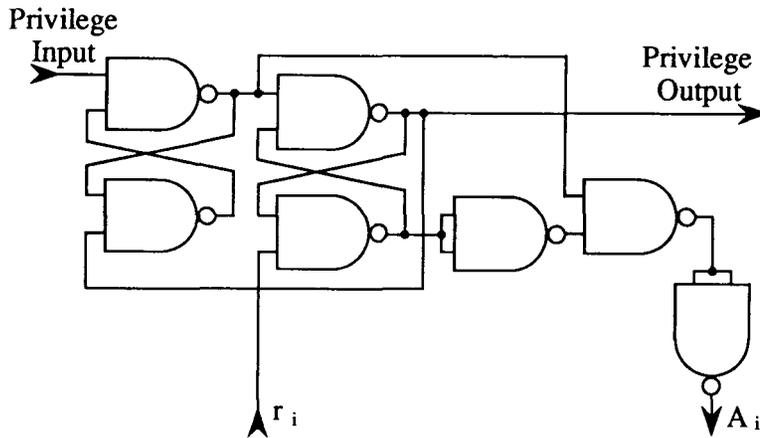


Fig. 2 Circuit of a cell.

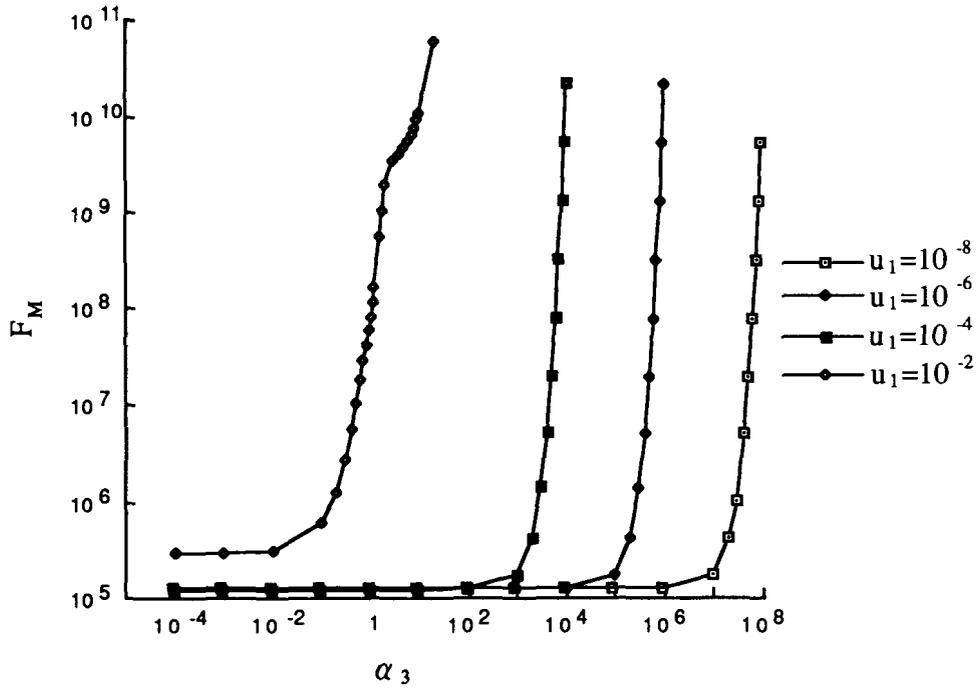


Fig. 3 Examples of calculated MTBF ( $T_0 = 200(nsec)$ ).

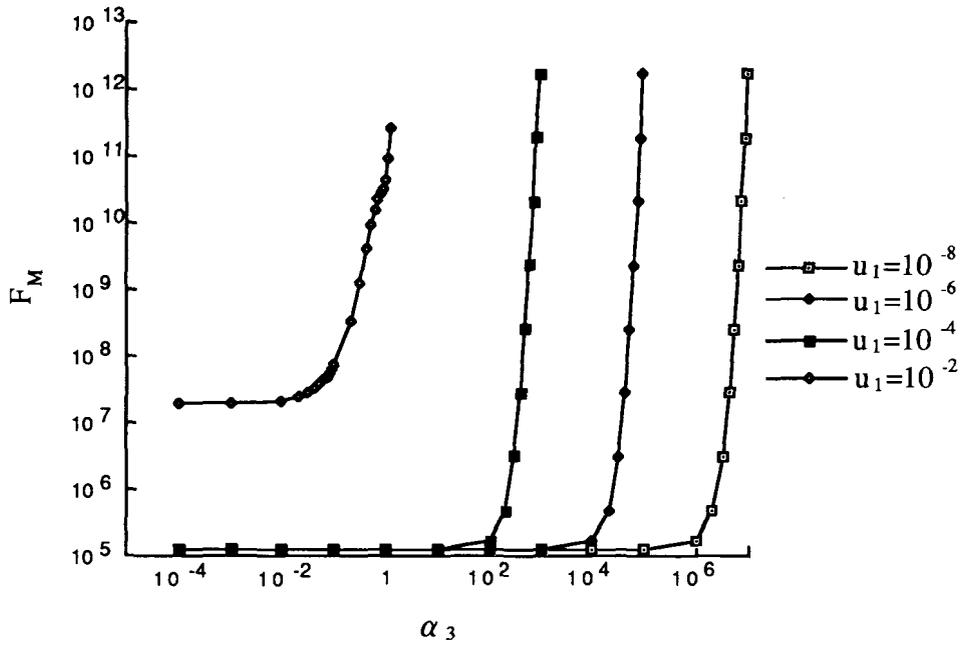


Fig. 4 Examples of calculated MTBF ( $T_0 = 300(nsec)$ ).

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