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A Procedure for Settling Multi-Criteria Problem by a Small Group of Decision Makers

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SYNOPSIS

AHP can handle decision-making problems involving several criteria when some of these are difficult or impossible to compare other than numerically. When a small group of decision makers settle the multi-criteria problem by AHP, the members of group could not often reach an agreement with hierarchic structure involving objectives, criteria, sub-criteria and alternatives because they have different positions, interests, and opinions. Further, the members have different importances for criteria and sub-criteria, and have dissimilar preferences for alternatives. In this article, we reveal the troubleness of AHP in case of being used by a small group of decision makers. Moreover, we proposed a procedure of AHP which the members of group could easily agree with the structure of problem and the weights of criteria etc.

1 Introduction[1)]

Frequently, an enterprise has to determine a new product, which should be developed, and a family has to select a durable consumer goods, which should be bought. On the occasion, there are some multi-criteria decision-making problems which can't be dealt with by a traditional mathematical programming method bacause criteria are difficult or impossible to compare other than numerically. The Analytic Hierarchy Process (AHP) which was developed by Thomas L. Saaty on the later 1970's is a useful method for these decision-making problems.

By the way, the members of group, who must settle these problems, have generally different positions and interests. Hence, they could not often reach an agreement with a hierarchy and they have different importances for criteria and sub-criteria, and have dissimilar preferences for

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alternatives. A procedure of AHP, which enables us to get over the troubleness for these decision-making problems, has to be established in order to reach the agreement and to get satisfying results.

As you have already known, the following devices had proposed for the members of group to use AHP smoothly.

1. A questionnaire table was designed which assisted for decision-makers to judge easily and the members were supported to express the result of pairwize comparison.
2. Exhibiting the results each other in order to be able to know the other member's opinion.
3. When the members could not reach an agreement for the results of pairwize comparison, they take the geometric-mean of individual judgements.

But, for only these contrivances, it is not easy to perform systematicly decision-making with the agreement among the members. Because some arguments still remain for the procedure when the members of group use AHP. For example, we have to examine how to construct a hierarchy which is used in common in the group, and how to deal with a troubleness that the judgements are not in agreement when some of members, who are compared with each other, have great influence.

In this article, first, we exhibit a procedure of constructing a hierarchy which consider opinions of every member as much as we can, as far as maintaining the independence of assessment criteria. Next, when weights for criteria are not agreed with each other, we propose a method that taking the weighted-geometric mean according to the influence of members. Moreover, we present a procedure of AHP for a small group of decision makers introduced these counterplan. And we consider a problem which type of facsimile is selected in a family, and we investigate the usefulness of this procedure.

2 The fundamentals of AHP[1)]

2.1 How to use AHP

A procedure of AHP for a single decision-maker are as follows;

1. Identifying and organizing decision objectives, criteria, sub-criteria and alternatives into a hierarchy.
2. Evaluating pairwize comparisons between the relevant elements at each level of the hierarchy.
3. The synthesis using the solution algorithm of the results of the pairwize comparisons over all the levels.

In next section, we illustrate the pairwize-comparison method which is the algorithm of computing the weights for criteria or alternatives.

2.2 Pairwize-comparison method

Let us consider the factors(criteria or alternatives) $x_1, x_2, \cdots, x_n$, and their weights $w_1, w_2, \cdots, w_n$. We assume that a decision-maker estimated the preference ratio $\frac{w_i}{w_j}$ to be a_{ij}. A pairwize-comparison matrix which have a_{ij} as ith row and jth column factor is represented to be A. The matrix A has the next form ideally.

$$A = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \tag{1}$$

We multiply the matrix A on the right by the weight-vector $w=(w_1, w_2, \cdots, w_n)^T$.

$$\begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} \tag{2}$$

It was shown that n is an eigenvalue of A and w is a right eigenvector of A with eigenvalue n. Even if A is not to be as Eq.(1) actually, it was regarded as Eq.(1), when we calculated an eigenvalue of pairwize-comparison matrix and an eigenvector of it, we may consider that each component of eigenvector is a relative weight of each factor.

Now, we assume that actual pairwize-comparison matrix is A', largest eigenvalue is $\lambda_{\max}$ and right eigenvector is v, then

$$A' = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1 \end{bmatrix} \tag{3}$$

and

$$A'v = \lambda_{\max} v \tag{4}$$

If $a_{ik}=a_{ij} \cdot a_{jk}$ for all i, j, k $(1 \leq i, j, k \leq n)$, we call the matrix A consistent and $\lambda_{\max} = n$. But generally $\lambda_{\max} \geq n$ (the proof is omitted), because the sum of eigenvalues of the matrix A' is n, $\lambda_{\max} - n$ is an index which indicate the size of eigenvalue without $\lambda_{\max}$. A' have n eigenvalues including $\lambda_{\max}$, so $\frac{\lambda_{\max} - n}{n-1}$ is the mean value of eigenvalues for A' without $\lambda_{\max}$. Saaty named it consistency index(C.I.) which is an indicator "closeness to consistency". If C.I.$\leq$0.1, we may be satisfied with our judgements and we can use each component of eigenvector with the largest

eigenvalue as relative weight of each factor. If C.I.>0.1, the judgements must be revised. It is good for us to state the verbal judgements (equally important, weakly more, strongly more, demonstrably more, absolutely more) first and then we translate them into their numerical values(NV).

3 A procedure of AHP for a small group of decision makers[2)3)4)]

In case that a family has to select a durable consumer goods, there are often a number of decison-makers. Under such circumstances, reaching the agreement by almost members is more desirable than executing by leader alone. But following obstracts may happen when the members of group use AHP, because they have different positions and interests.

1. Some of the members are blind to AHP.
2. When the members construct a hierarchy, they may not reach the agreement with a hierarchy involving objectives, criteria, sub-criteria and alternatives.
3. The members have different opinions for the numerical values alloted to modifiers for representing the result of pairwize-comparison.
4. When some of the members have greater influence than other, it is difficult to reach the agreement with scale values.

We present a procedure of AHP by a small group of decision makers which is contrived to get over these troubleness (Figs.1 to 4). Subsequently, we illustrate steps of the procedure.

Fig.1 explains the whole outline of AHP by a small group of decision makers.

Fig.2 shows the preparation of AHP by a small group of decision makers. Decision makers may not know certainly what a pairwize-comparison method is, but they have to know how to use AHP. The members have often different opinions for the scale values. In case some of the members have greater influence than other, it is not rational to take a (simple) geometric mean. Taking weighted-geometric mean according to their influence etc. helps to reduce their discontent, and will leads to the agreement for the conclusion of the problem. If NV, which apply to the result of pairwize-comparison, differ with the members, it is inconvenient for their judgements to be synthesized. It is need for the members of group to use NV in common. To be concrete, a member who knows AHP well shows several NV, each member may select NV which suits his feeling among these NV, or may create a new one with the referrence to them. Of course, these values must be satisfied with a cardinal consistency. Nothing can be better than the agreement of the members with NV used in common as a result of discussion. But if decision makers could not reach the agreement with NV, they could calculate the new NV, which the members of group use

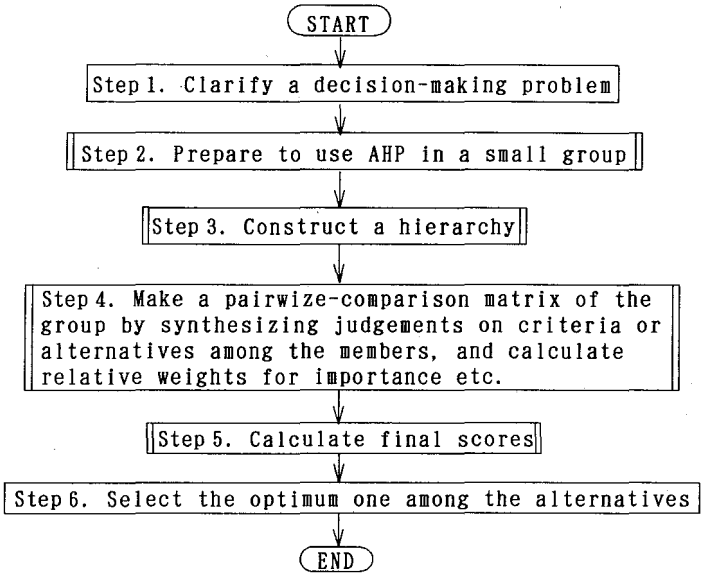

Fig. 1 The outline of AHP by a small group of decision makers

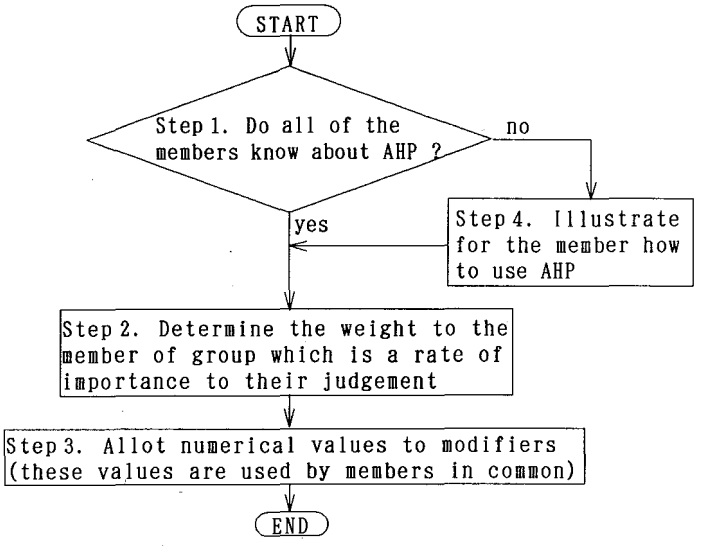

Fig. 2 Preparation for decision-making in AHP

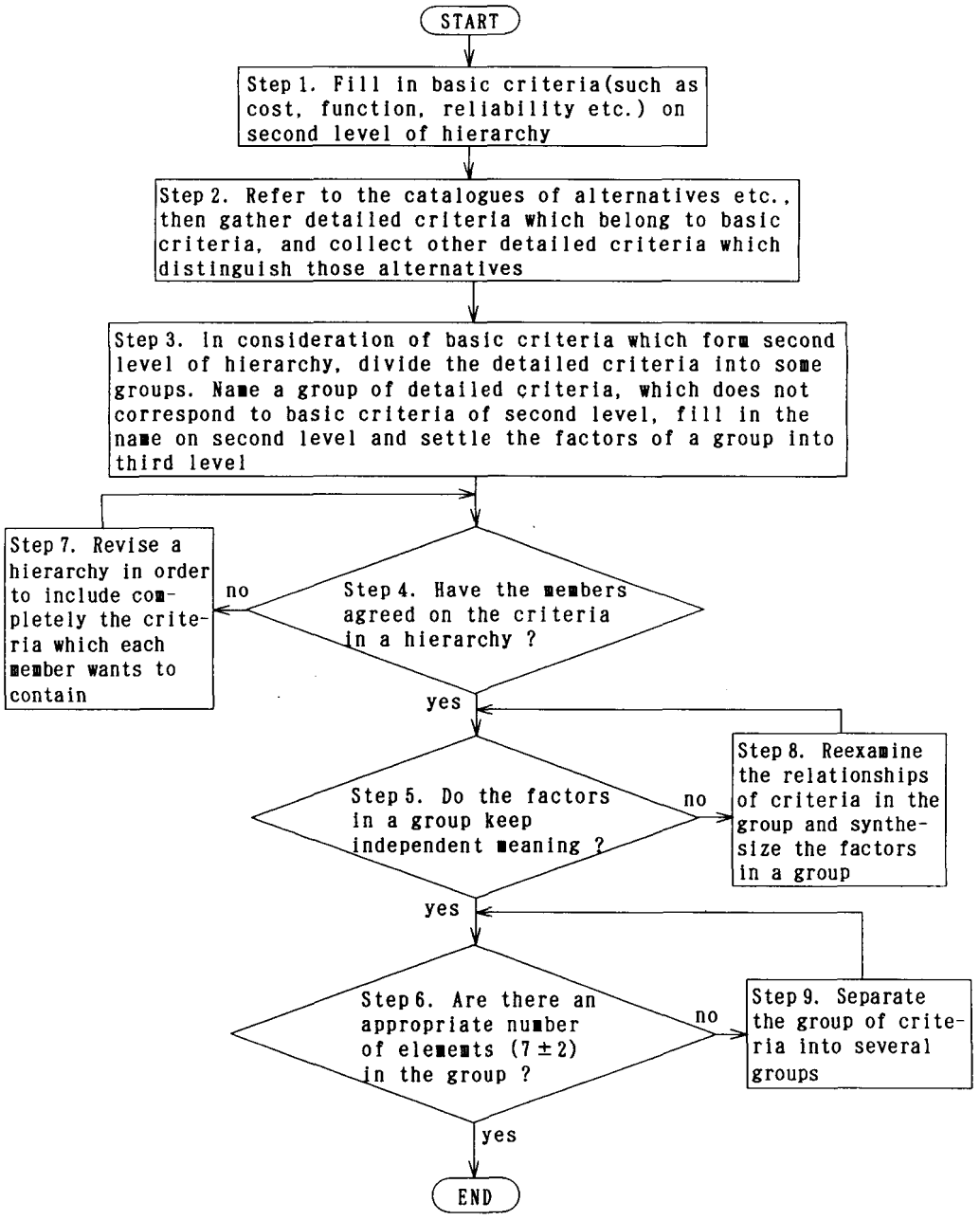

Fig. 3 A step of constructing a hierarchy

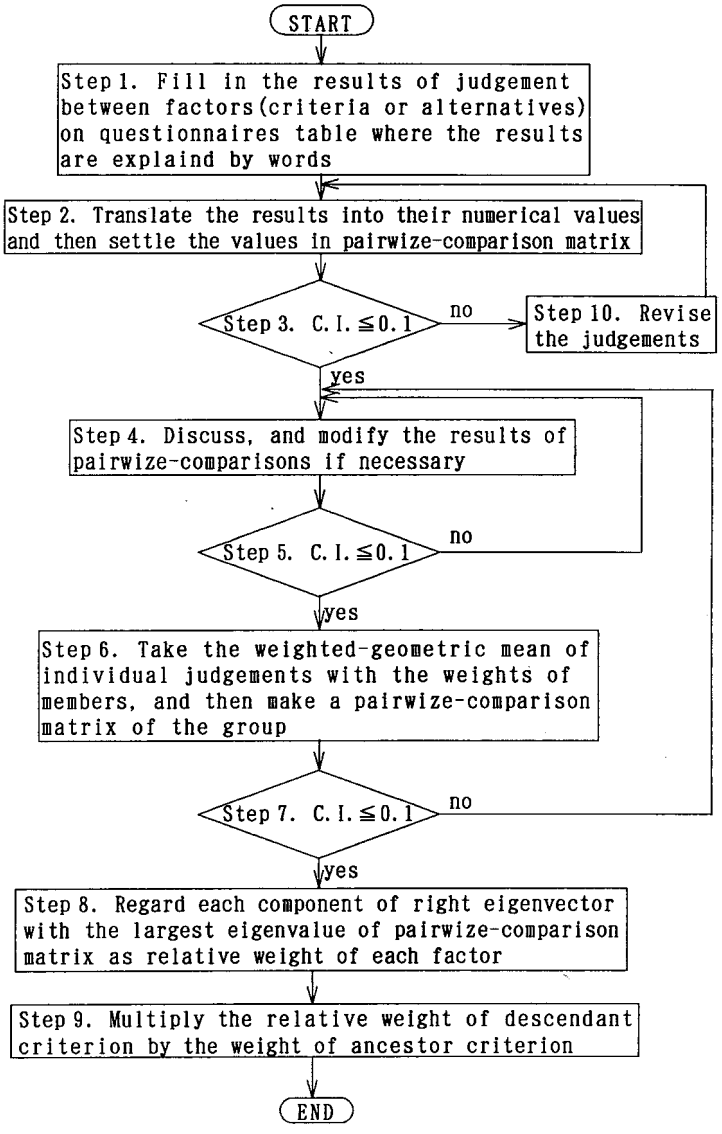

Fig. 4 A step of calculating relative weights

in common, by taking the (non-weighted) geometric mean of numerical values which the members alloted for the same modifiers. In this step, weighted-geometric mean hasn't been taken. If we make weighted-geometric mean for the numerical values, it is used double, with a step of making the result of pairwize-comparison of the group.

Fig.3 elucidates how to construct a hierarchy. According to the treated problem, there are some differences in the step of constructing a hierarchy. When it purposes to select a durable consumer goods, criteria which form second hierarchy level have almost been decided(cost, function, reliability etc.). We first enter basic criteria in second level of hierarchy, next fill in detailed criteria, which are related each other, under second level. As mentiond above, we could construct a hierarchy. We have to examine a hierarchy until the following facts are all satisfied. 1)The members have agreed on the criteria in a hierarchy. 2)The factors in a group keep independent meaning. 3)There are an appropriate number of elements (7 ± 2) in the group.

Fig.4 expounds the way to calculate final scores of group. First, the results of pairwize-comparison between factors are entered in a questionnaire table(see table 1). Next, the group calculate C.I. of a pairwize-comparison matrix. If C.I.$\leq$0.1, they can make a discussion with the judgements shown each other. The members of group should ask to a member which gave an extreme judgements, why he gave these judgements. That member might misunderstand about factors or be aware of the point where the others overlooked. If the members of group have different opinions for the scale values after going through due discussion, they should take weighted-geometric mean according to the rate which was decided in step of preparation. As mentioned above, we have made a pairwize-comparison matrix of the group.

4 Selecting the most suitable facsimile for a family

The facsimile is a communication machine which can transmit similar figures of the original picture by using a communication circuit. When a family use a facsimile, several people(such as Father, Mother, and children)use it in common. Because they use facsimile in various ways, they have respectively different expectation for the facsimile. Ill feeling may be possibly remained if the members of family had not agreement with the selection of facsimile. On this ground, we consider a problem which type of facsimile is selected in the family, then with this example, we illustrate a procedure of AHP by a small group of decision makers, which is proposed in chapter 3, and we investigated the usefulness of it. A small group of decision makers consists of three members, such as Mr.O, Mr.F (collage students) and Mr.S(a bank clerk) who uses facsimile frequently. They selected suitable four alternatives in many facsimile after going through due discussion in consideration of function and cost. These alternatives are shown in Table 2.

Now we have to determine the weights of the members which is a rate of importance to their

Table 1 An example of questionnaire table to make a pairwize-comparison matrix (It was judged for the 'Fuction' by Mr. O)

is absolutely more important than	is demonstrately more important than	is strongly more important than	is weakly more important than	is equally important as	is weakly less important than	is strongly less important than	is demonstrately less important than	is absolutely less important than	
Communication management			×						Expand transmission
〃				×					Preservation of security
〃				×					Image
〃						×			Cope with outside line
Expand transmission				×					Preservation of security
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Image						×			Cope with outside line

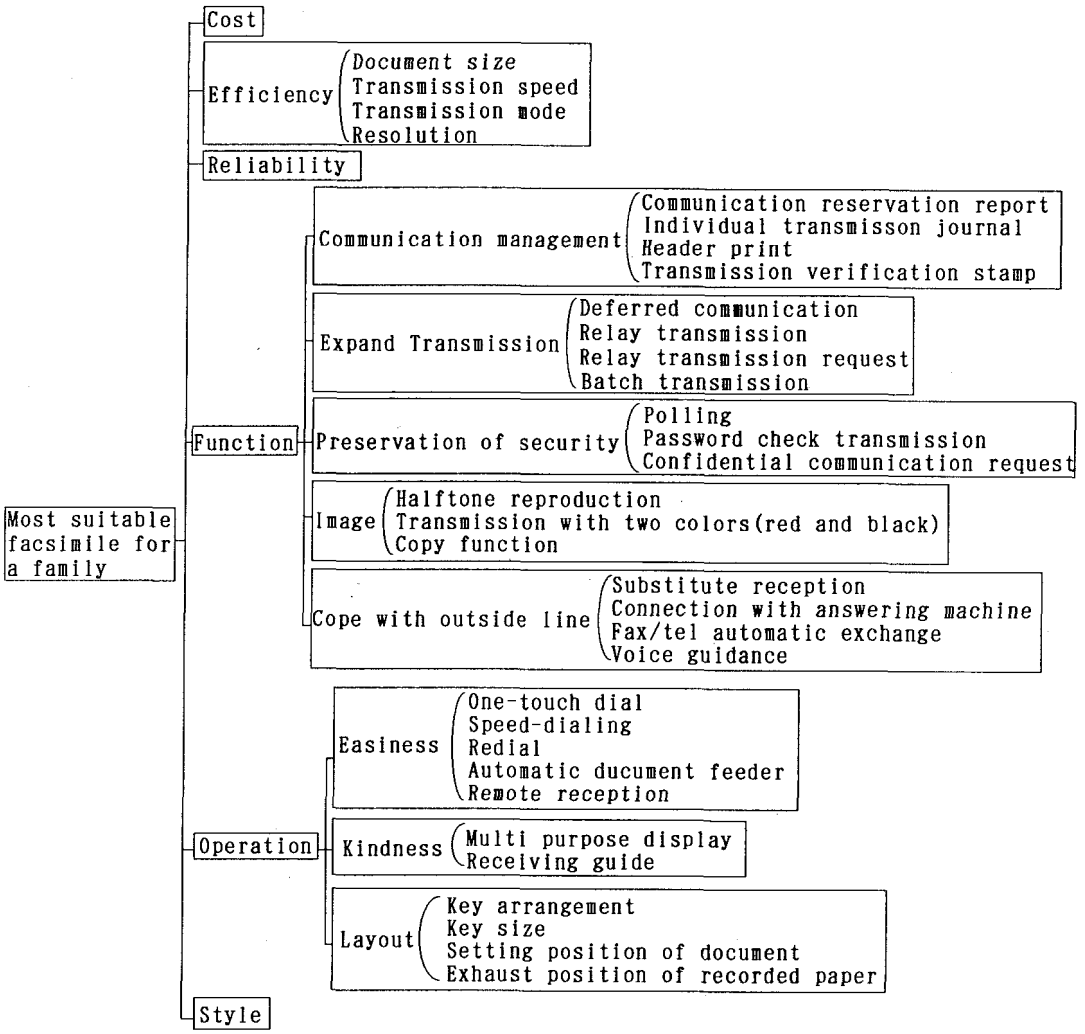

Fig. 5 A hierarchy to select a facsimile for a family

Table 2: Cost and features of alternatives

Type and Cost	Features
Type A ¥118,000	High cost performance and simple operation. A4 size transmission. Connection with answering machine.
Type B ¥148,000	Transmission with two colors(red and black). Compact A4 size transmission.
Type C ¥168,000	Super fine mode for clean and sharp transmission. High speed 15-second B4 size.
Type D ¥278,000	Involving answering machine. Communication reservation report. Multi purpose display. High speed 15-second B4 size.

Table 3: A pairwize-comparison results for the weights of the members

	Mr.O	Mr.F	Mr.S
Mr.O	Equally important	...	...
Mr.F	Equally important	Equally important	...
Mr.S	Strongly more important	Strongly more important	Equally important

judgements. In this article, they discussed about the weights and reached an agreement with Table 3.

Pairwize-comparisons are translated into numerical values which are shown in Table 4. We can calculate an eigenvector with the largest eigenvalue, v=(0.2 0.2 0.6)^T. An importance ratio of attached to their judgement gets Mr.O:Mr.F:Mr.S=1:1:3. The result reflects that Mr.S uses facsimile so frequently and knows it well. We assume that a certain scale values of pairwize-comparison results are 1(Mr.O), 1.7(Mr.F), 3(Mr.S), then a scale value of the group is $\sqrt[5]{1^1 \times 1.7^1 \times 3^3} = 2.15$. We may calculate a scale value of the group on every criterion respected for the member's opinion in the favorite field. Criteria which form second hierarchy level in the selection of facsimile are shown in Table 5. We consider criteria which belong to each criterion in second hierarchy level and arrange, then we have constructed a hierarchy(Fig.5).

In the next step, the members must give the weights with criteria and alternatives. We

Table 4: Numerical values alloted to modifiers for representing the result of pairwize comparison

Modifier		Values
Equally important	→	1
Weakly more important	→	1.7
Strongly more important	→	3
Demonstrably more important	→	10
Absolutely more important	→	100

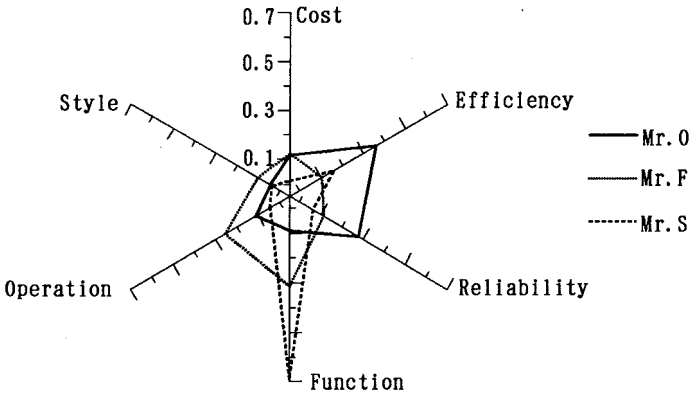

Fig. 6 Relative weights of second hierarchy level

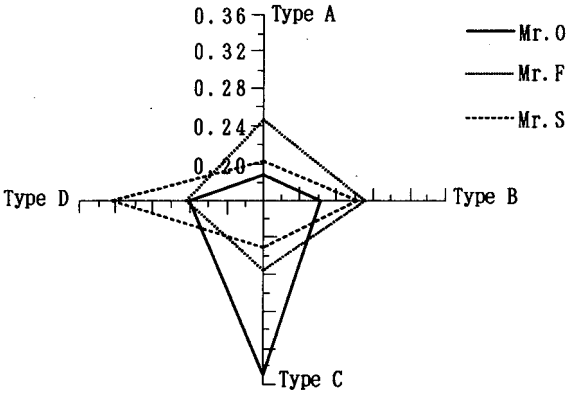

Fig. 7 Final scores of each member for alternatives

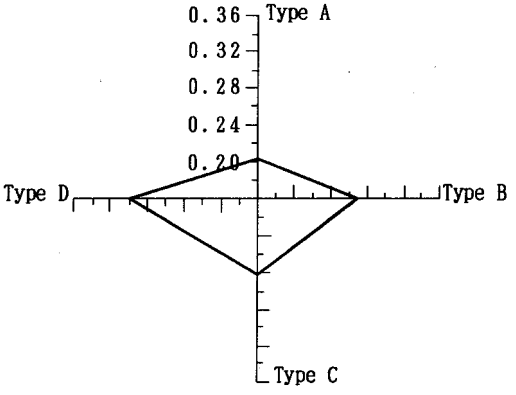

Fig. 8 Final scores of the group for alternatives

Table 5: Criteria which form second hierarchy level to the problem

Group	Contents
Group a	Cost of alternatives.
Group b	Basic function of alternatives.
Group c	Reliability and safety of alternatives.
Group d	Additional function which enables us to use facsimile more effectively.
Group e	Style of alternatives.

compare relatively the weights between each pair of two factors, and fill the relative weights in a questionnaire table. When whole comparison are completed, we translate them into their numerical values, and then we have made a pairwize-comparison matrix. Fig.6 shows relative weights of second hierarchy level of the members for this example. Fig.7 gives final scores of each member for alternatives. Fig.8 indicates final scores of the group for alternatives.

Fig.6 express that each member has his own preference for the criteria. The result displayed that an alternative, which was given the highest priority, differed with the members. It is difficult to select the suitable alternative with their preferences accepted in such a condision. We calculated final scores by using a proposed procedure, Type D is the most preferred facsimile for this example. The result signified that the intention of Mr.S was directly reflected as it was. But each alternative, which Mr.O and Mr.F gives the highest priority, is not by far the best, and they give second importance to Type D. So these members consented that Type D, which was selected according to final scores of the group, was purchased.

5 Conclusion

In this article, we proposed a procedure for settling multi-criteria problem by a small group of dicision makers. As we construct a hierarchy in this procedure, the members could enough inconporate our criteria and alternatives in the tree. Moreover, we may maintain the independence of them. When we systhesize our judgements for the weight of importance, scale values of the group are determined by discussion as much as we can. If the members could not reach an agreement with scale values after going through due discussion, we take the weighted-geometric mean according to our experience or power, and the members adopt it as the group results, then we have arrived at consensus. As a result of applying this procedure to the facsimile-selection problem, each member understood the others opinions and we could select a desirable alternative systematicly.

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