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An Adaptive Observer via Optimal Control Law

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Synopsis

This paper deals with the adaptive observer which estimates the states and parameters of unknown system. It is shown that the adaptive observer problem is reduced to the identification of the transformation matrix for an arbitrary designable observer. Moreover, the adaptive process of the unknown parameters is reduced to the linear optimal regulator problem. As the result, a new method is presented to obtain an appropriate adaptive process with good insight.

And, in this identification, a linear filter is found to be also useful against noises in input-output data. To achieve high accuracy, a particular nonlinear filtering can improve SN ratio only in the direction of the unknown vector. Even if SN ratio of input-output data has zero dB, sufficient accuracy can be accomplished within suitable correction time.

This design algorithm seems to be rather straightforward and practical. Since input sequence is required to be only sufficiently general, the method is applicable to on-line identification also.

1. Introduction

In the linear control problems, an observer frequently serves for an state estimator. Although a priori knowledge about the object

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system must be required for the observer design, it seems frequent in the practical situations that the object system contains some unknown parameters or varying ones.

The adaptive observer [1,2,3] is well known as a device for such problems. Especially, the adaptive observer based on stability theory , proposed by Lüders et al. [2,3], is useful and its correcting process for unknown parameters is guaranteed to be asymptotically stable in the large. But, although its convergence speed crucially depends on arbitrarily selectable parameters, any method to select these parameters is not discussed at all. In the practical designing, it is very difficult to choose such parameters appropriately.

We reduce the adaptive observer problem into the identification problem of observer transformation matrix with the canonical form similar to one of Lüders et al. And we propose the method which solves an adaptive law for parameter estimation as the optimal regulator problem for the linear system whose state vector is composed of estimation errors of unknown parameters.

Though it is necessary to choose proper performance index (P.I.) so as to obtain a desirable solution for optimal regulator problem, in this problem, we can determine proper P.I. with consideration of noise involved in input-output data and computation error.

In the sequel, an appropriate solution can be found in rather straightforward manner in the practical synthesis.

Furthermore, we believe that this method is also useful for the estimation of sampled impulse response and other identification problems.

First, in section 2, we formulate the problem and reveal that the adaptive observer problem can be reduced into the identification problem of an observer transformation matrix. In section 3, correcting algorithm of unknown parameters is formulated. Then, we show that an appropriate adaptive law is found as the solution of optimal regulator problem. And we discuss the effective preliminary data processing against noise in input-output data. In section 4, computer simulation for some examples is illustrated and confirms the validity of this method. Finally, summary and discussion of the results in this paper are in section 5.

2. Problem Formulation and Reduction to Identification

Object unknown system is supposed to be a n-th order linear discrete time system. We assume that the system is completely observable and controllable.

It is well known that such a system can be represented, generally, as (1) with arbitrarily selected observable pair (F,r) [2,3]. Lüders et al. call the equation similar to (1) as the canonical form for an adaptive observer. In the following discussion, we will deal with the system represented by the difference equation (1).

$$\begin{cases} x_{k+1} = \left[a \,\middle|\, \begin{matrix} r^T \\ \hline F \end{matrix} \right] x_k + b u_k \\ y_k = (1, 0, \cdots, 0) \cdot x_k = x_k^1 \end{cases} \tag{1}$$

where k is integer and denotes time. x_k, u_k and y_k are n×1 state vector, scalar input and scalar output at k-th time, respectively. For simplification, we suppose that r is column vector of (n-1)-th order and $F = \mathrm{diag}(\lambda_2, \lambda_3, \ldots, \lambda_n)$. $(.)^T$ denotes transpose. Besides, input sequence $\{u_k\}$ is supposed to be sufficiently general.

The problem considered here is to construct an adaptive scheme with stable and satisfactory convergence for the estimation of x_k and identification of the pair of unknown vector $\{a,b\}$ under above assumptions.

Generally, the initial values x_0^i of x_k^i ($i \in \underline{n}$) are unknown except $x_0^1 = y_0$. Let the initial values be $\bar{x}_0 \triangleq (x_0^2, x_0^3, \ldots, x_0^n)^T$ and we will regard these as a part of unknown vector to be identified.

Now, consider an another system of (2). The relationship of (3) always holds between z_k, w_k and x_k.

$$\begin{cases} \begin{bmatrix} y_{k+1} \\ \bar{z}_{k+1} \end{bmatrix} = \left[r \,\middle|\, \begin{matrix} a^T \\ \hline F \end{matrix} \right] \cdot \begin{bmatrix} y_k \\ \bar{z}_k \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \left(b^T \begin{bmatrix} u_k \\ \bar{w}_k \end{bmatrix} + r^T F^k \bar{x}_0 \right), \\ \bar{w}_{k+1} = F \bar{w}_k + r u_k , \\ w_k \triangleq (u_k, \bar{w}_k^T)^T , \quad z_k \triangleq (y_k, \bar{z}_k^T)^T , \\ \bar{w}_0 = \bar{0} , \quad \bar{z}_0 = \bar{0} , \quad y_0 = y_0 \end{cases} \tag{2}$$

$$\begin{cases} x_k^i = a_i \cdot z_k^i + b_i w_k^i + x_0^i \lambda_i^k \quad ; \quad i \in \underline{n} , \ k = 1, 2, \cdots \\ x_k^1 = z_k^1 = y_k \quad ; \quad k = 1, 2, \cdots \end{cases} \tag{3}$$

Define c, $\hat{c}_k$ and s_k as follow.

$$\begin{cases} c \triangleq (a^T, b^T, \bar{x}_0^T)^T , \\ \hat{c}_k \triangleq (\hat{a}_k^T, \hat{b}_k^T, \hat{\bar{x}}_{0k}^T)^T , \\ s_k \triangleq (z_k^T, w_k^T, r^T F^k)^T \end{cases} \tag{4}$$

When (c, s_k) denotes inner product of c and s_k, the first row of (2) is rewritten as below.

$$y_{k+1} = (c, s_k) \quad ; \quad k = 1, 2, \cdots \tag{5}$$

where $\{s_k\}$ and $\{y_{k+1}\}$ are accessible data, c is unknown parameter vector to be estimated and $\hat{c}_k$ is estimate vector of c. (5) is in the similar form to a fundamental equation appearing in identification problem of sampled impulse response. So, we can obtain a successive estimate $\hat{c}_k$ of c in some suitable way.

Let the estimate of x be $\hat{x}_k$. When we will employ the elements of $\hat{c}_k$ corresponding to a_i, b_i, $x_0^i (i \in \underline{n})$, the resulted system denoted by (2') is found to be an observer for (1) asymptotically (or adaptively).

From the study of Luenberger [4], any stable system whose driving inputs are the input and output of another one may be an observer for the latter and the transformation matrix which satisfies such a relation does almost always exist. Therefore, the adaptive observer problem can be interpreted as the identification one for the transformation matrix.

3. Adaptive Scheme for Unknown Parameters

Several procedures to obtain the estimate $\hat{c}_k$ of unknown vector c using (5) often appear in the general identification problem such as one for sampled impulse response etc. Accordingly, various identification techniques are available.

3.1 Discussion about Conventional Identification Scheme

The identification scheme proposed by Lüders et al. is summalized as below;

First, to evaluate $\hat{y}_{k+1}$ in (6) and utilize (7) for sequential adaptive scheme for the estimate $\hat{c}_k$.

$$\hat{y}_{k+1} = (\hat{c}_k , s_k) \tag{6}$$

$$\hat{c}_{k+1} = \hat{c}_k + P_k \cdot (y_{k+1} - \hat{y}_{k+1}) \cdot s_k \tag{7}$$

Taking into account (5) and (6) and introducing error vector $\phi_k \triangleq c - \hat{c}_k$, (7) is reformed as (8).

$$\phi_{k+1} = (I - P_k \cdot s_k \cdot s_k^T)\, \phi_k \tag{8}$$

Next, to choose a stable diagonal matrix as P_k and select its elements so that satisfactory convergence will be achieved. However, its selection principle is not proposed at all. Furthermore, determination of them causes great difficulty in the practical synthesis.

Following consideration clarifies above situation. Now, (8) is considered as a linear state feedback control system of order m (m=3n-1) whose state vector is estimate error ϕ_k. If $(-P_k \cdot s_k \cdot s_k^T \phi_k)$ is regarded as the control input via state feedback, then the number of arbitrarily assignable eigenvalues of matrix $(I-P_k \cdot s_k \cdot s_k^T)$ is always one. Namely, if (7) is employed as correcting law, the dimension of controllable subspace of the system in (8) must be one. It means that there never exists, generally, any method to determine P_k which guarantees convergence speed.

Furthermore, the learning identification method [5] which often serves for identification of sampled impulse response is identical with the case of $P_k \triangleq (\alpha/\|s_k\|)\cdot I$, $(0<\alpha<2)$ in (7).

From observation of (8), all the (m-1) eigenvalues of $(I-P_k \cdot s_k \cdot s_k^T)$ equal to one and remaining one eigenvalue turns to be $p=(1-\alpha)$. If $0<\alpha<2$, then $-1<p<1$. Therefore, only one eigenvector component corresponding to p will be corrected and other components remain uncorrected. Moreover, the eigenvector corresponding to p is proportional to s_k.

Consequently, it is guaranteed that $\|\phi_k\|$ decreases monotonously as k increases, but its convergence speed crucially depends on the signal vector $\{s_k\}$.

When the Gramian of the set of successive m vectors $\{s_{k-m+1}, \ldots, s_k ; (k=m,m+1,\ldots)\}$ has a sufficiently large value, above correcting method must be effective. In the adaptive observer problem, however, it is expected from the generating scheme of $\{s_k\}$ that the Gramian has very small value. Therefore, the convergence becomes extremely poor after some correcting stage.

Only from the point of view of convergence speed, we already reported a method which converges $\|\phi_k\|$ to zero after finite count (theoretically m-steps) corrections using Gram-Schmidt orthogonal method. But, this method has a fatal disadvantage to additive noise in input-output data and will not be useful in practice [6].

In the identification problem for impulse response, a parameter correcting scheme such that its convergence speed is guaranteed in some degree relative to observation noise, has been reported [7]. But , since this scheme employs the special signal (M-sequence) as identification input, it does not seem to be applicable for this case.

Another method [8] was reported, which partially introduced Gram-Schmidt orthogonalization, as an extension of the scheme proposed by Nagumo et al.(for the sake of impulse response identification).

When the number of orthogonal vectors used at each correction time becomes large, this method will have a fatal disadvantage against the noise as well as the method employing Gram-Schmidt.

3.2 New Parameter Correcting Scheme

New correcting scheme proposed in this paper allows to determine appropriately the convergence speed of error vector ϕ_k to the origin.

Consider the controllable subspace of the system whose state vector is ϕ_k. This scheme employs a correction formula so that it raises the dimension of the subspace up to m. And then, it sets each eigenvalue of the system to suitable amount.

As the correction formula to this end, for example, (9) may be derived as an extension of (7).

$$\hat{c}_{k+1} = \hat{c}_k + \sum_{i=1}^{m} P_{ki} \cdot (y_{k-i+2} - \hat{y}_{k-i+2}) \cdot s_{k-i+1} \tag{9}$$

where

$$\hat{y}_{k-i+2} = (\hat{c}_k \, , \, s_{k-i+1}) \quad ; \; i \in \underline{m} \, , \; k = 1, 2, \cdots \tag{10}$$

If $P_{ki} \triangleq (1/\| s_{k-i+1} \|) \cdot I$ is selected, then $P_{ki} \cdot (y_{k-i+2} - \hat{y}_{k-i+2}) \cdot s_{k-i+1}$ is proven to be the orthogonal projection of error vector $\phi_k = c - \hat{c}_k$ onto s_{k-i+1}. As the result, by selecting P_{ki} like this for all $i \in \underline{m}$, we can know the coordinate of ϕ_k in the vector space spanned by the set of $\{ s_{k-i+1} \; ; \; i \in \underline{m} \}$. Therefore, if $\{ P_{ki} \}$ is multiplied by suitable transformation matrix, ϕ_k vanishes with arbitrary speed.

In the similar manner to (8), (9) is modified into the formula with respect to error vector ϕ_k with (5).

$$\phi_{k+1} = (I - \Gamma_k) \cdot \phi_k \tag{11}$$

where

$$\Gamma_k = \sum_{i=1}^{m} P_{ki} \cdot s_{k-i+1} \cdot s_{k-i+1}^T \tag{12}$$

When input sequence $\{ u_k \}$ is sufficiently general, the set of signal vectors $\{ s_{k-m+1}, \ldots, s_k \; ; \; (k=m, m+1, \ldots) \}$ is expected to be relatively independent and it is possible to choose P_{ki} ($i \in \underline{m}$, $k=m, m+1, \ldots$) such that rank $\Gamma_k = m$. So, all the eigenvalues of $(I - \Gamma_k)$ are arbitrarily assignable.

Now, the problem is how to choose suitable Γ_k for the scheme. If we set all the eigenvalues of $(I - \Gamma_k)$ far enough in the left half plane, convergence speed may increase but insensitivity against noises will become poor. Well, (11) is interpreted as a closed loop system with feedback gain Γ_k. By setting proper P.I., solution for the optimal regulator problem will present a desirable pole allocation. This problem is parallel to determination of Kalman Filter gain[9]. Therefore, we can design an adaptive scheme with taking into account the noise variance etc.

From above discussion, we conclude that optimal adaptive law is reduced to optimal regulator problem.

Now, suppose that consideration of additive noise in the input-output sequence and computation error determines the performance criterion and that Γ_k is presented as the optimal control law for that criterion.

We discuss how to determine P_{ki} satisfying (12).

If it is required only for rank Γ_k to be m , the later terms in (9) s_{k-i+1} ($i \in \underline{m}$) are not essential. Therefore, an arbitrary independent vector set is allowed so as to determine P_{ki} simply and we can replace s_{k-i+1} with m-th order column vector e_i which has value one only at the i-th element; i.e.,

$$e_i \triangleq (0, \cdots, 0, \overset{i}{1}, 0, \cdots, 0)^T$$

We will deal with resulted equation (9') in the following discussion.

$$\hat{c}_{k+1} = \hat{c}_k + \sum_{i=1}^{m} P_{ki} \cdot (y_{k-i+2} - \hat{y}_{k-i+2}) \cdot e_i \tag{9'}$$

Though (9') does not reveal the physical meaning so clear as (9), we can make rank Γ_k to m and reduce the optimal adaptive law to the optimal regulator problem. In parallel to (11), we construct the equation about error vector ϕ_k.

$$\phi_{k+1} = (I - \Gamma_k) \cdot \phi_k \tag{11'}$$

where

$$\Gamma_k \triangleq \sum_{i=1}^{m} P_{ki} \cdot e_i \cdot s_{k-i+1}^T \tag{12'}$$

If we construct the matrix P_k by extracting the i-th column of each P_{ki} ($i \in \underline{m}$), p_k^i, in turn, i.e.,

$$P_k \triangleq (p_k^1, p_k^2, \cdots, p_k^m) \tag{13}$$

(12') is rewritten as (14).

$$P_k \cdot S_k = \Gamma_k \tag{14}$$

where $$S_k \triangleq (s_k, s_{k-1}, \cdots, s_{k-m+1})^T \tag{15}$$

Therefore, (16) determines the matrix P_k, i.e., each i-th column of matrix P_{ki} ($i \in \underline{m}$), p_k^i.

$$P_k = \Gamma_k \cdot S_k^{-1} \tag{16}$$

It is clear from (9') or (12') that by dint of post multiplication of e_i, only i-th column p_k^i in the matrix P_{ki} can affect the result and the result is independent of another columns. Consequently, practical correction is carried out in accordance with (17).

$$\hat{C}_{k+1} = \hat{C}_k + \sum_{i=1}^{m} p_k^i \cdot (y_{k-i+2} - \hat{y}_{k-i+2}) \tag{17}$$

where p_k^i is i-th column vector of P_k in (16) and $\hat{y}_{k-i+2}$ is given in (10).

Next, we discuss how to determine the optimal control law Γ_k. As the performance criterion, following (18) is employed.

$$J = \frac{1}{2} \| \phi_N \|_{Q_1}^2 + \frac{1}{2} \sum_{k=0}^{N-1} \left(\| \phi_k \|_Q^2 + \| \Delta C_k \|_R^2 \right) \tag{18}$$

where

$$\Delta C_k \triangleq \hat{C}_{k+1} - C_k$$

Q_1, Q and R are symmetric positive definite matrices. Since state equation is represented as (19), it is well known that ΔC_k to minimize J in (18) is given as (20) [10].

$$\phi_{k+1} = I \cdot \phi_k + I \cdot \Delta C_k \quad ; \quad k \in \underline{N} \tag{19}$$

$$\Delta C_k^{\circ} = - R^{-1} \cdot (M_k - Q) \cdot \phi_k \quad ; \quad k \in \underline{N} \tag{20}$$

where M_k is the solution of matrix difference equation.

$$M_k = Q + M_{k+1} \cdot (I + R^{-1} M_{k+1})^{-1} , \quad M_N = Q_1 \tag{21}$$

Accordingly, optimal control law Γ_k is represented as (22).

$$\Gamma_k = R^{-1} (M_k - Q) \quad ; \quad k \in \underline{N} \tag{22}$$

In the practical synthesis, if the limiting value of M_k as $N \to \infty$ in (18), is adopted , then Γ_k becomes constant about sampling time k and it may be convinient to determine Γ_k in (16).

3.3 The Case of Noise Presence

When we can not ignore the noise in input-output data, we may devise the following two countermoves in roughly classification.

1) Structure which is possibly least insensitive for noise influence.
2) Equivalent improvement of variance ratio of signal to noise (i.e., SN ratio) for input-output data.

As a device 1), to select an element of F appropriately so that the signal vector s_k does not deviate in a direction or to select P.I. , equivalently optimal control law Γ, so as to reduce the correction magnitude at a time.

Since the effect of either method is limiced, another device 2) will be demanded. As a countermove of 2), data processing on input-output with a suitable linear filter may be employed so as to improve SN ratio. Namely, as illustrated in Fig.1, we take the filter with same characteristics (pulse transfer function $G(z^{-1})$) on the contaminated input-output data $\{u_k\}$, $\{y_k\}$ and deal with the filtered sequence $\{\tilde{u}_k\}$, $\{\tilde{y}_k\}$.

Now, we suppose that input-output sequences $\{u'_k\}$ and $\{y'_k\}$ are contaminated by uncorrelated white gaussian noises $\{n_k\}$ and $\{m_k\}$ whose averages are zero and variances are σ_n^2 and σ_m^2, respectively and denote these sequences $\{u_k\}$ and $\{y_k\}$ as in Fig.1.

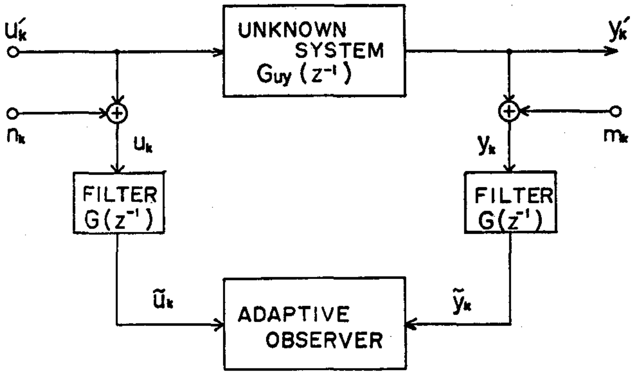


Fig.1 A structure of an adaptive observer using a linear filter for noises

(23) is the basic equation for parameter estimation corresponding to (5).

$$y_{k+1} = (s_k, c) + v_k \quad ; \ k = 1, 2, \cdots \tag{23}$$

where

$$\begin{cases} v_k \triangleq m_{k+1} - (\xi_k, \bar{c}) \ , \quad \bar{c} \triangleq (a^T, b^T)^T \\ \xi_k \triangleq (m_k, {}_m\bar{Z}_k^T, n_k, {}_n\bar{W}_k^T)^T \\ {}_m\bar{Z}_{k+1} \triangleq F \cdot {}_m\bar{Z}_k + \Gamma m_k \quad , \ {}_n\bar{W}_{k+1} = F \, {}_n\bar{W}_k + \Gamma n_k \end{cases} \tag{24}$$

From (23) and (24), $\{v_k\}$ has correlation to $\{y_{k+1}\}$ and $\{s_k\}$ but its mean is zero. When we denote the tilde value as the input-output sequence through the filter $G(z^{-1})$, (23) is rewritten as (25).

$$\tilde{y}_{k+1} = (\tilde{s}_k, c) + \tilde{v}_k \quad ; \ k = 1, 2, \cdots \tag{25}$$

For example, if we take the first order linear filter $G(z^{-1})=1/(1-gz^{-1})$, (25) is represented as (25').

$$\sum_{i=0}^{k} g^{k-i} \cdot y_{k-i+1} = \left(\sum_{i=0}^{k} g^{k-i} \cdot s_{k-i}, c \right) + \sum_{i=1}^{k} g^{k-i} \cdot v_{k-i} \tag{25'}$$

Design of the filter $G(z^{-1})$ is based on a priori information about signal and noise, and it is well known in circuit theory.

Next, advancing the utilization of the linear filter, we propose the more effective improvement method for SN ratio. It is, first, to add several data in the form of (22), considering their signs in left hand side, and then to employ the resulted equation (26) instead of (5).

$$y^*_{k+1} = (s^*_k, c) + v^*_k \quad ; \ k = 1, 2, \cdots \tag{26}$$

where

$$\begin{cases} y^*_{k+1} \triangleq \sum_{i=1}^{p} |y_{k-p+i+1}| \ , \quad s^*_k \triangleq \sum_{i=1}^{p} \mathrm{sgn}(y_{k-p+i+1}) \cdot s_{k-p+i} \ , \\ v^*_k = \sum_{i=1}^{p} \mathrm{sgn}(y_{k-p+i+1}) \cdot v_{k-p+i} \quad ; \ k = 1, 2, \cdots \end{cases} \tag{27}$$

This method is interpreted as an employment of a sort of filter. While, in case of linear filter, $\{y_{k+1}\}$ and each element of $\{s_k\}$ are simply improved about the SN ratio of $\{y_{k+1}\}$ to $\{v_k\}$, viz. SN ratio along only c direction is effectively improved. In the latter case, we may regard s_k, y_{k+1} and v_k as input signal, response at that time and noise, respectively.

As the geometric interpretation, see Fig.2. When the vector space is divided into two subspaces by a superplane orthogonal to vector c (one including c called positive half space). Data processing with (26), which superposes the left hand side in (23) all positively, means to shift the signal vectors $\{s_k\}$ into positive half space and to make the c-directional component increase. Practically, however, in the presence of noise $\{v_k\}$, there are some signal vectors which are not shifted into positive subspace correctly. Nevertheless, it is expected that even if noise $\{v_k\}$ has arbitrary large magnitude against $\{y_{k+1}\}$, probability of correct shifting will be greater than one-half.

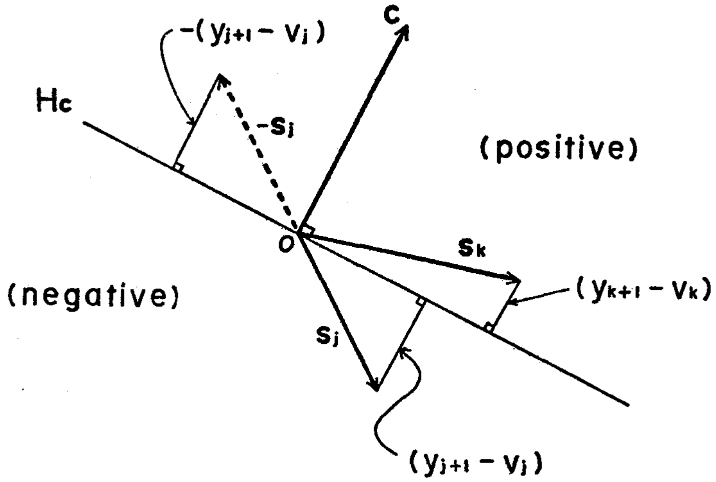


Fig.2 An illustration of transposition of signal vectors $\{s_k\}$, (k = 1, 2,..)

4. Computational Examples

In order to illustrate the effectiveness of the adaptive observer proposed in this paper, computer simulations are carried out for some examples.

The object system is described in the observable canonical form by (28).

$$\begin{cases} x_{k+1}^{o} = \begin{bmatrix} 0 & 0 & 0.5507 \\ 1 & 0 & -2.0222 \\ 0 & 1 & 2.4670 \end{bmatrix} x_k^{o} + \begin{bmatrix} 0.0716 \\ -0.1726 \\ 0.1036 \end{bmatrix} u_k \\ y_k = (0 \quad 0 \quad 1)\, x_k^{o} \end{cases} \tag{28}$$

Though input sequence is arbitrary provided to be sufficiently general, we employ the sum of six sampled sinusoidal waves which have relatively different frequencies so that each frequency ratio is irrational and have appropriate phase shifts and amplitudes. Average and variance of input sequence $\{u_k\}$ are zero and one, respectively.

4.1 Time Invariant Unknown Parameters

First, we examine the case which the parameters to be estimated are constant and noises in input-output are negligible small. Here, we take F in (1) as F = diag(-0.9, -0.95) and initial value as $\bar{x}_0$ = $(10.0,10.0)^T$. As performance criterion in (18), let $N \rightarrow \infty$ and

Q = diag(1.0,1.0,...,1.0) ,

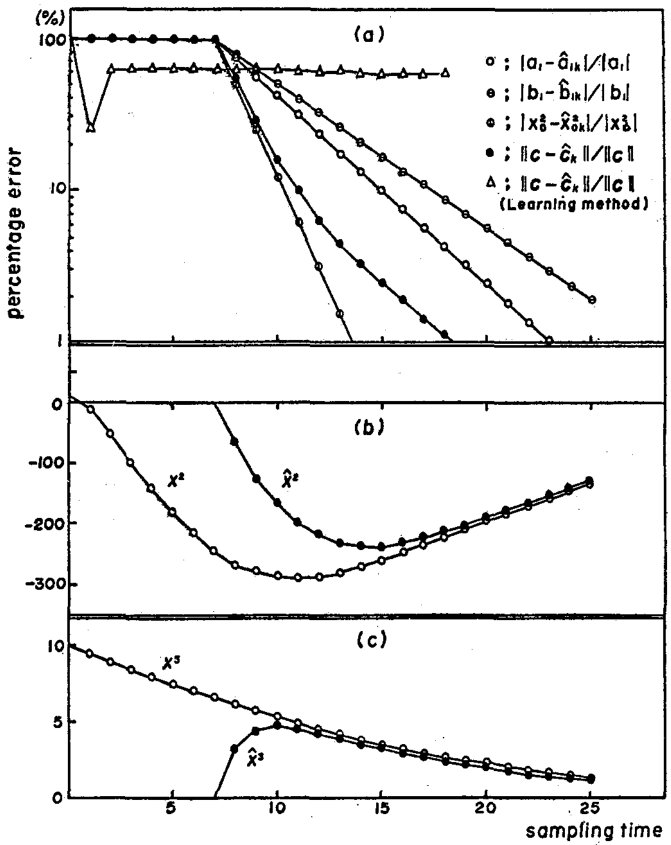
R = diag(12.0,12.0,12.0,20.0,20.0,20.0,2.0,2.0) .

Then, it follows that optimal control law is

Γ = diag(0.25,0.25,0.25,0.20,0.20,0.20,0.50,0.50) .

The results of correcting process for unknown parameters with adaptive law of (17) and of estimation process for state variables with (3) are both illustrated in Fig.3. Fig.(a) shows the convergence characteristics. (In the figure, only $\hat{a}^1_k$, $\hat{b}^1_k$ and $\hat{x}^2_{0k}$ are represented, however, with consideration of Γ, the characteristics of $\hat{a}^i_k$, $\hat{b}^i_k$ and $\hat{x}^i_{0k}$ will be plotted similarly, respectively.)

At a glance of these figures, it follows that unknown parameters follow faithfully to optimal control law and their convergence to the true values are stable and have desired speed, respectively.



Γ = diag [0.25,0.25,0.25,0.20,0.20,0.20,0.50,0.50]

(a) ; *Parameter estimation*

(b),(c); *State estimation*

Fig.3 Results of parameter and state estimation (In case of 4.1)

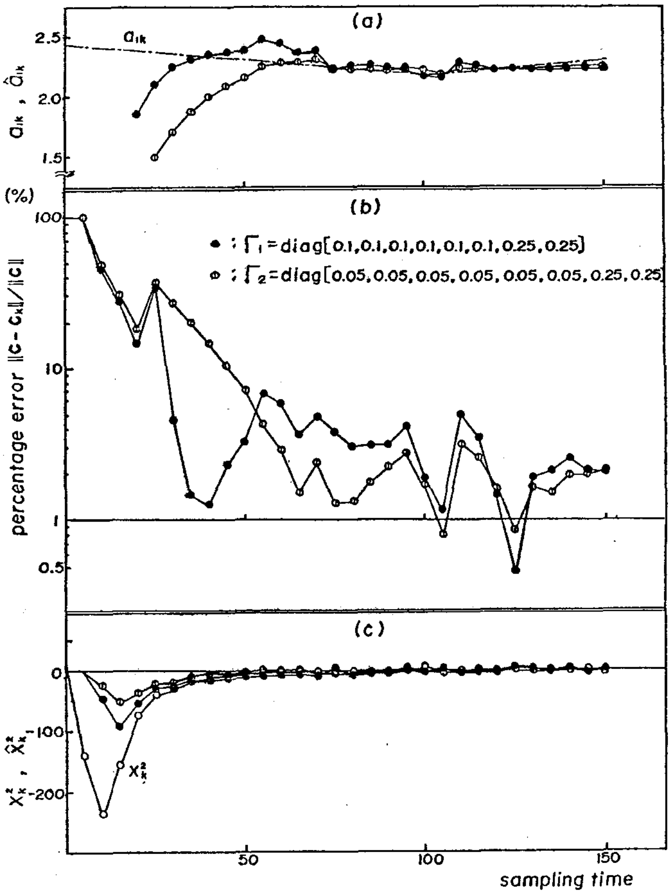
Fig.(b) shows the state estimation at this time. In the figure, symbol ▲ represents the correcting process by the learning identification method with gain α =1.0 and comparison with Fig.(a) suggests the effectiveness of the method proposed.

4.2 Time Variant Unknown Parameters

Next, the case which unknown parameters a' and b' vary in time with triangular wave form around the values employed in the previous section is examined for two type performance criteria. Here, the parameters except a' and b' and the input sequence are identical with ones in section 4.1.

Fig.4 shows the convergence process of the unknowm parameters and state variables. (In Fig. (a),(b) and (c), symbols ●,⦶ represent the result of the optimal contrl law Γ_1 and Γ_2 ,respectively and symbol o represents the true value.)

In the case of varying parameters, it is obvious that the smaller eigenvalues of $(I-\Gamma)$ does not necessarily follow the better result. Therefore, suitable selection of P.I. remains to be future study. However, the availability of this method is confirmed.



(a) ; Parameter estimation
(b) ; Convergence of error of estimation
(c) ; State estimation

Fig.4 Results of parameter and state estimation (In case of 4.2)

4.3 Noise Presence

For the same system and input sequence in section 4.1, a simulation is carried out for the case which input-output data $\{u_k\}$ and $\{y_k\}$ are contaminated by noises $\{n_k\}$ and $\{m_k\}$, respectively. Let F = diag (-o.95, -0.85) and $\bar{x}_0 = (1.0,1.0)^T$ and the initial value is not estimated

Here, we denote $\{n_k\}$ and $\{m_k\}$ as pseudo white gaussian process which have zero means and variance σ_n^2 and σ_m^2 , respectively and have no correlation one another.

Since we can not ignore the noise in this case, we employ the device for noise attenuation discussed in section 3.3 as preliminary procedure.

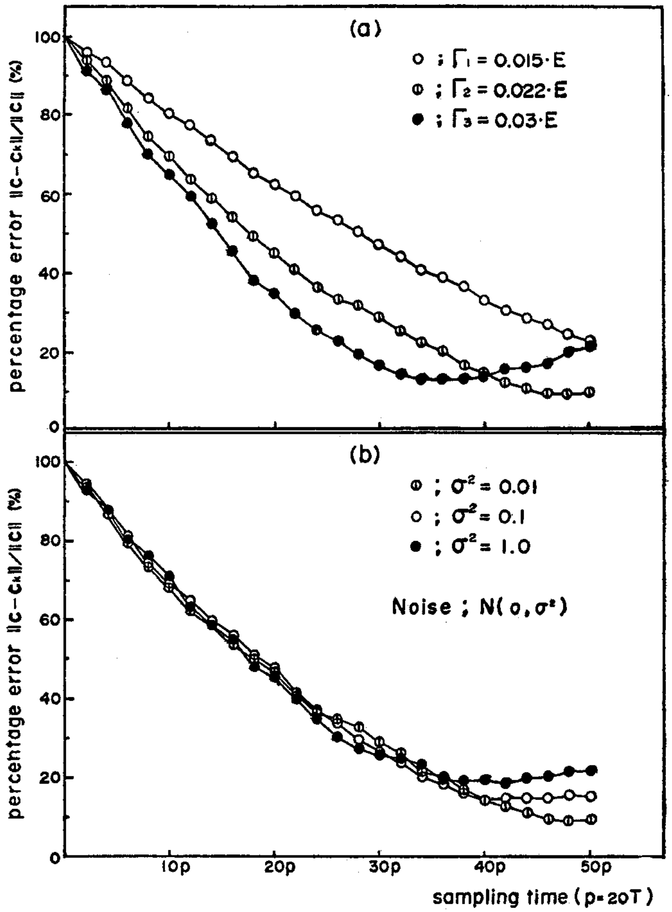
First, we take use of the first order filter $G(z^{-1})=1.0/(1.0-0.91z^{-1})$ (The cutoff frequency of this filter is set to be equal to the maximum frequency contained in the input sequence.) Data processing of (26) and (27) is performed for p = 20 .

Fig.5 shows the convergence process of unknown parameters.

In Fig.(a), we show the results of these types of criteria and of constant variance ($\sigma_n^2 = \sigma_m^2 = 0.01$). Conversely, Fig.(b) shows the case of fixed performance criterion and various variances of noise.

From Fig.(a) and (b), it follows evidently that if the final accuracy is taken into account seriously, it is necessary to keep correcting speed small enough. However, if we want to achieve as high accuracy as possible within allowable correcting time, it is necessary to seek as suitable optimal control law as possible.

After 20 times of simulation, the average value of $\|c-\hat{c}_k\|/\|c\|$ is plotted in the graph and the variance of the estimation error is of order $10^{-3} \sim 10^{-4}$.



(a) ; Variance $\sigma^2 = 0.01$ (Noise ; $N(o,\sigma^2)$)
(b) ; Control law $\Gamma = 0.022 \cdot E$ (E ; unit matrix)

Fig.5 Results of parameter estimation (In case of 4.3)

It is notable from these graphs that this method is useful against noises in input-output data with considerable magnitudes.

5. Conclusions

We reduce the problem of designing an adaptive observer for the linear discrete time system to the identification problem for the transformation matrix of an observer arbitrarily predetermined.

Moreover, we reduce the identification problem of unknown parameters to linear optimal regulator problem whose state vector is estimation error and propose a new parameter estimation method. And, as the device to reduce the influence of noises in input-output data, we show that we can take use of a linear filter in order to improve SN ratio of input-output data. And also, regarding noise in the signal vector as a part of the signal, we propose the method which effectively improve SN ratio of the data along only unknown vector direction.

Conventional adaptive observers based on stability theory do not guarantee convergence speed in parameter estimation. From the point of view of this speed, we show that our method presents a suitable correcting process under an appropriate performance criterion with consideration of noises in data and deviation of parameters.

And, we illustrate its usefulness by some numerical examples.

Since parameter correction in this method requires only sufficient generality for input sequence, it seems to be useful for on-line identification.

It is an important problem in the future to seek some rule on which we can suitably determine F that is arbitrary parameter on transforming a given system into a canonical form for an adaptive observer. It is not fully solved how to determine the weighting matrices in performance index. In the practical synthesis, however, it is known that careful observation of the system responses yields good insight in choosing these matrices.

It is under consideration whether such an idea can be applicable to the adaptive observer synthesis.

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